Mathematical Traditions and the Problem of Change

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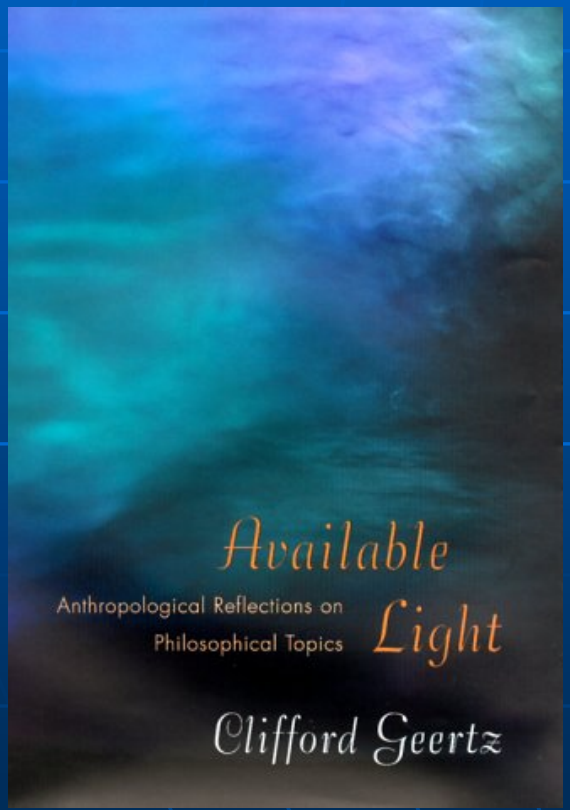
Staying with Tradition and Seeing Change

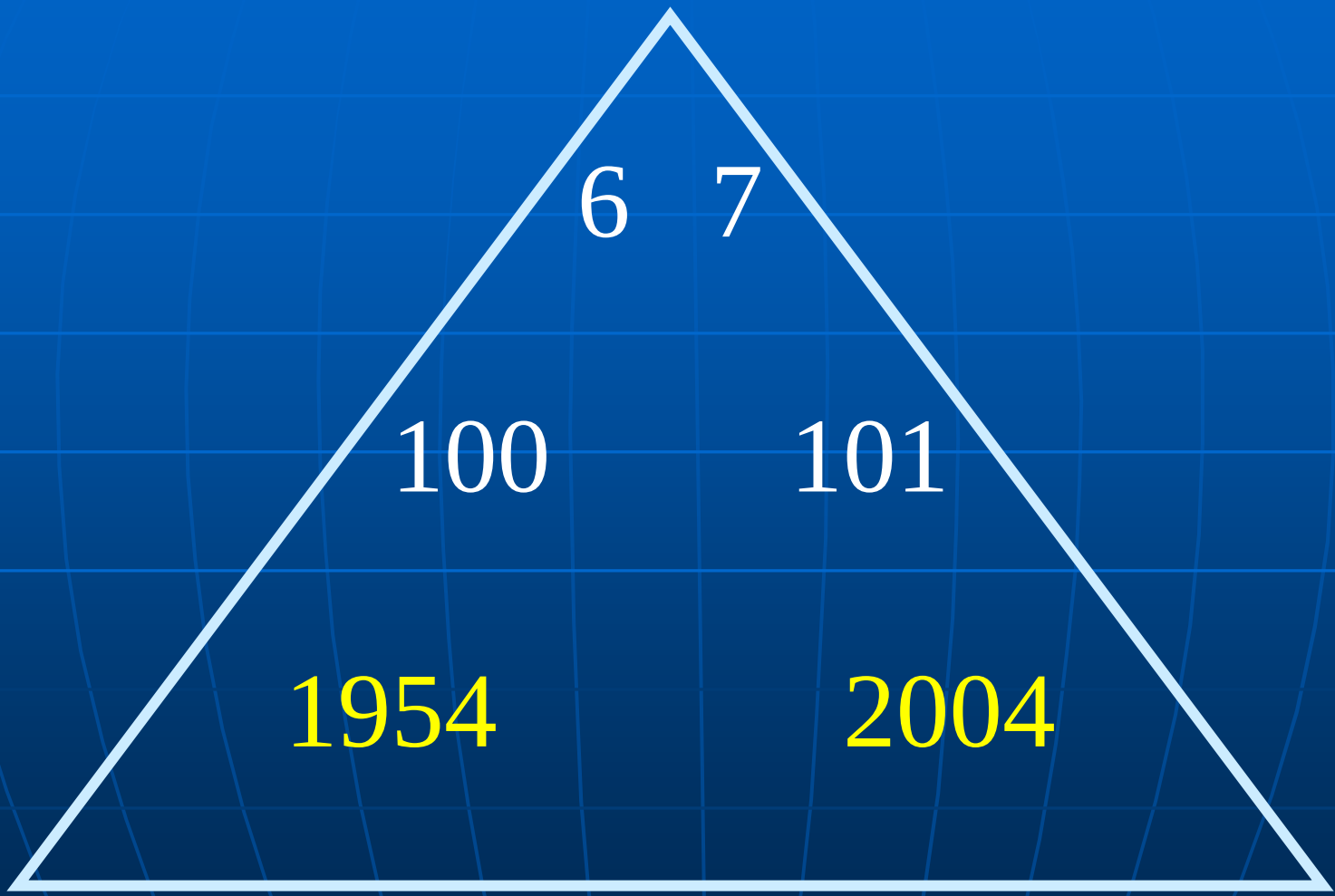
An Anthropologist's Advice



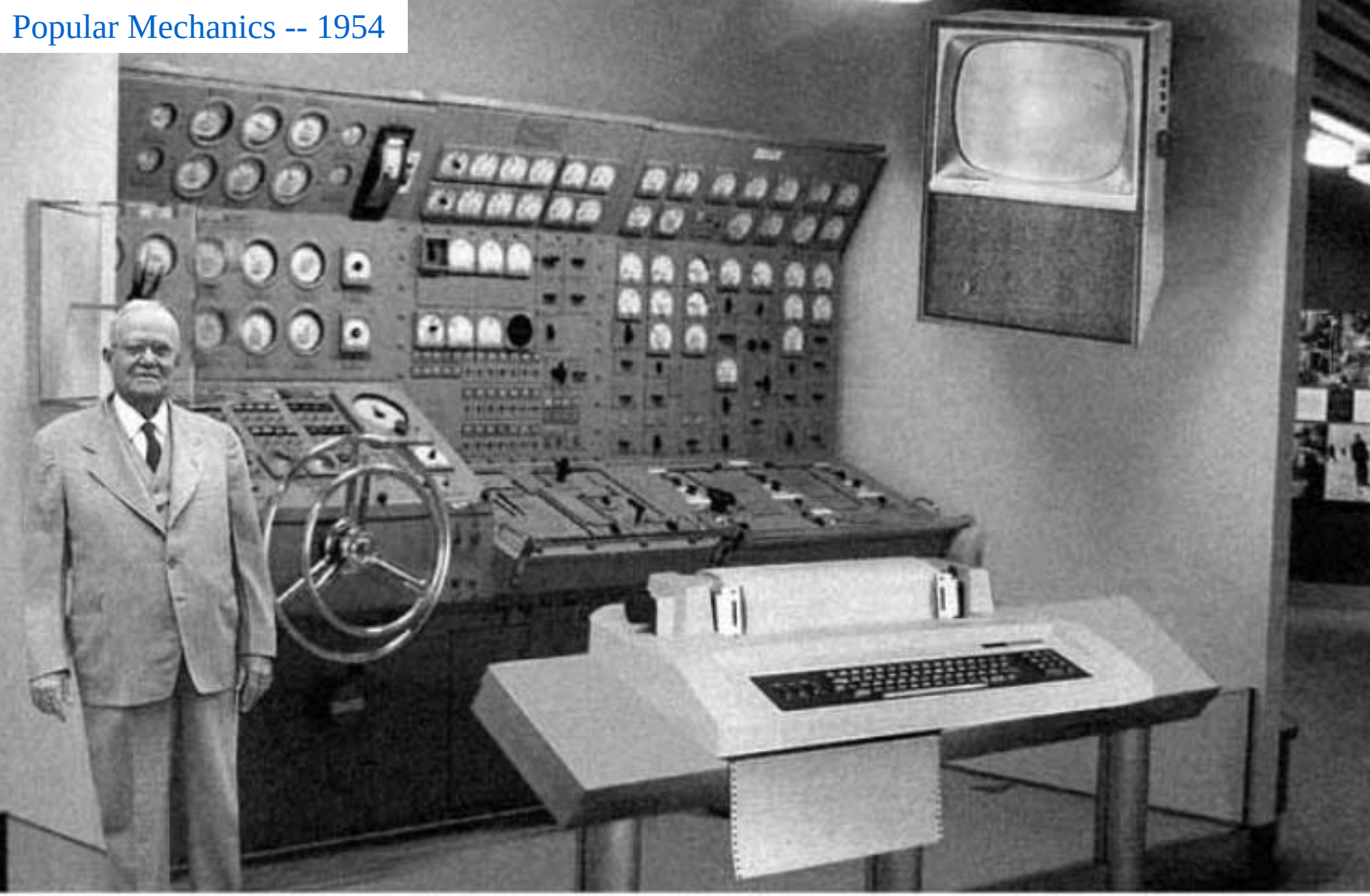
"… there are a good many more ways of getting it wrong than getting it right, and one of the most common ways of getting it wrong is through convincing ourselves that we have gotten it right …"

- Clifford Geertz

Off by One,
Off by More Than One

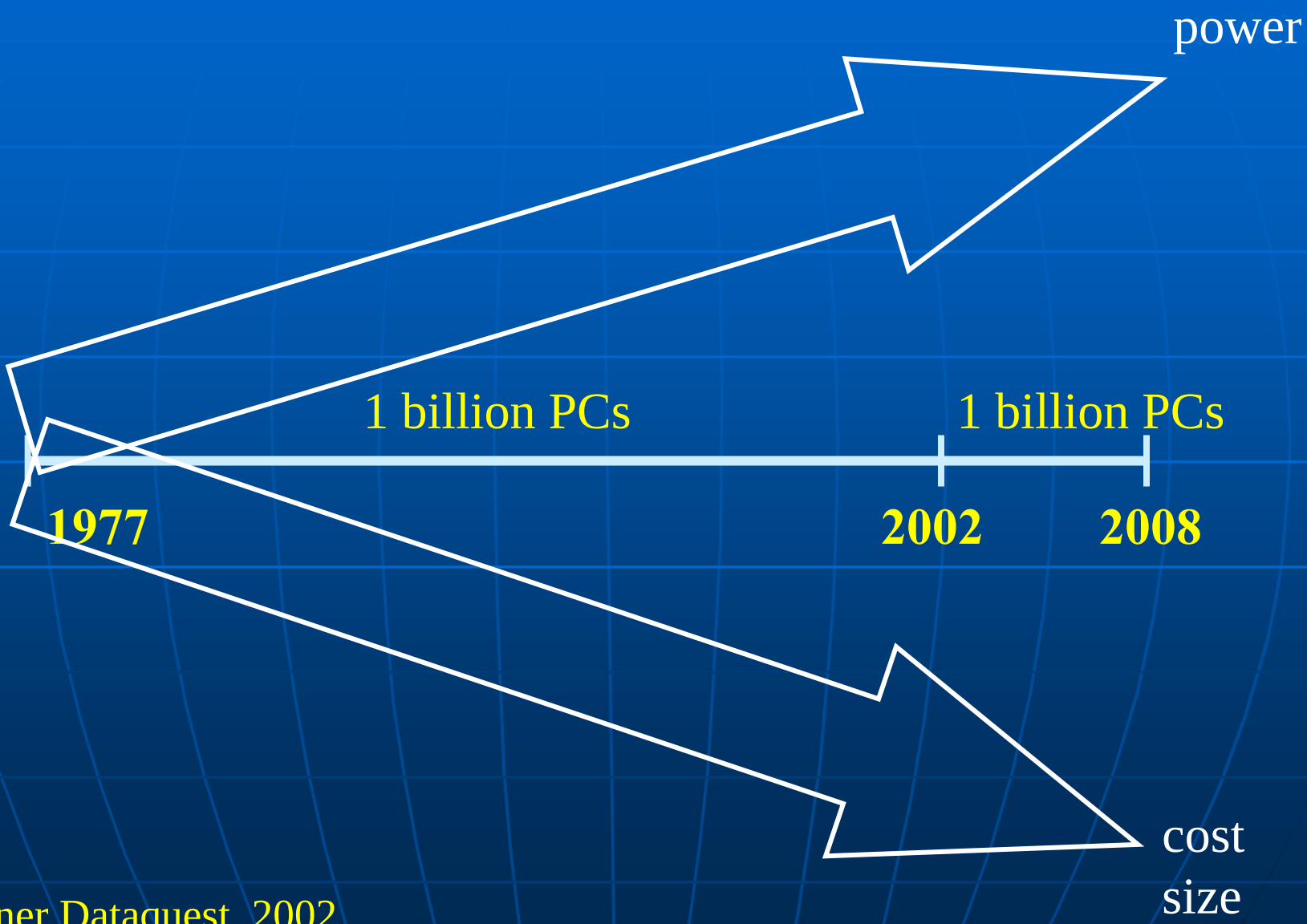


Popular Mechanics -- 1954



Scientists from the RAND Corporation have created this model to illustrate how a "home computer" could look like in the year 2004. However the needed technology will not be economically feasible for the average home. Also the scientists readily admit that the computer will require not yet invented technology to actually work, but 50 years from now scientific progress is expected to solve these problems. With teletype interface and the Fortran language, the computer will be easy to use.

The Impact of Technological Change

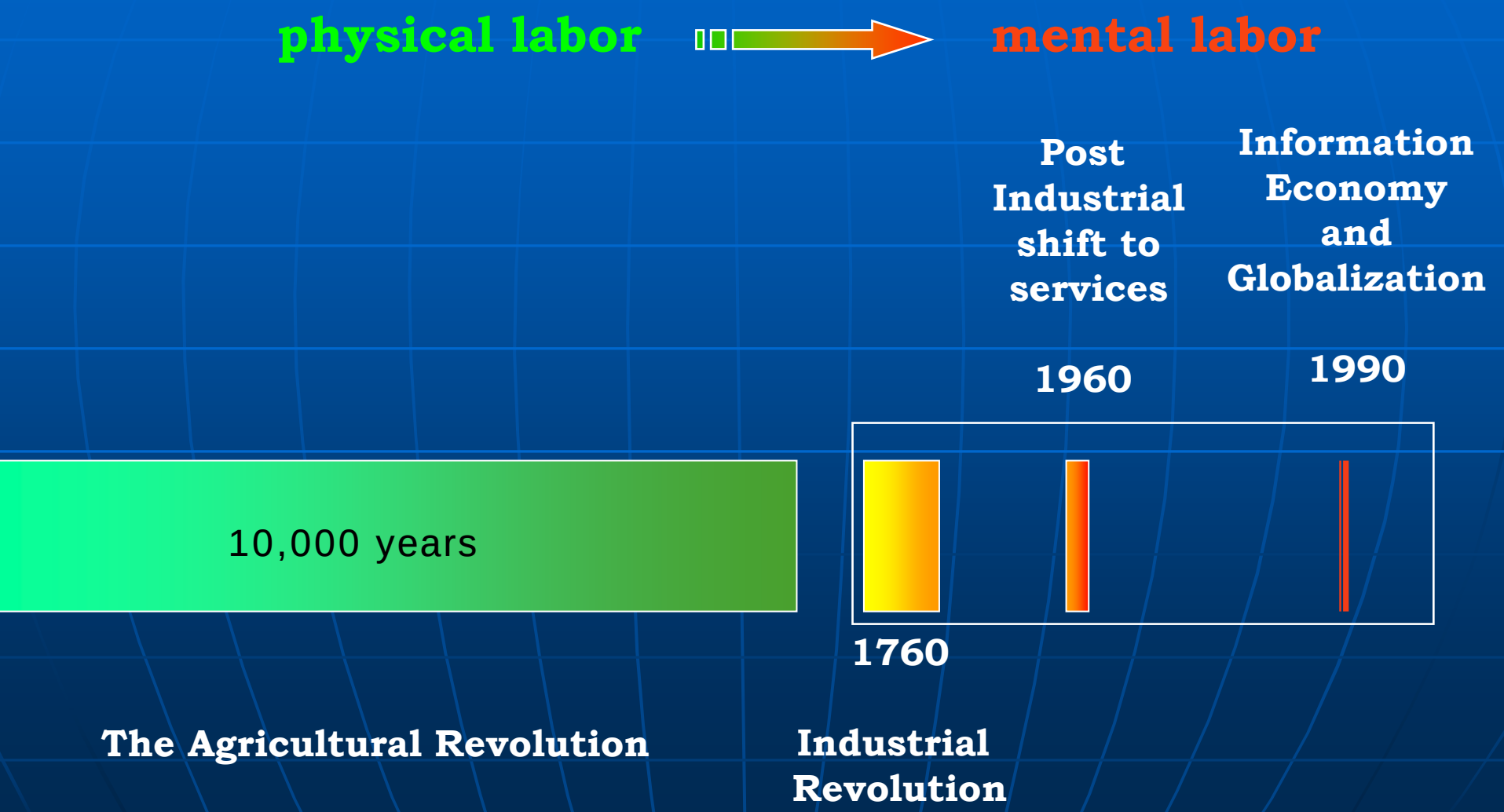


Gartner Dataquest, 2002

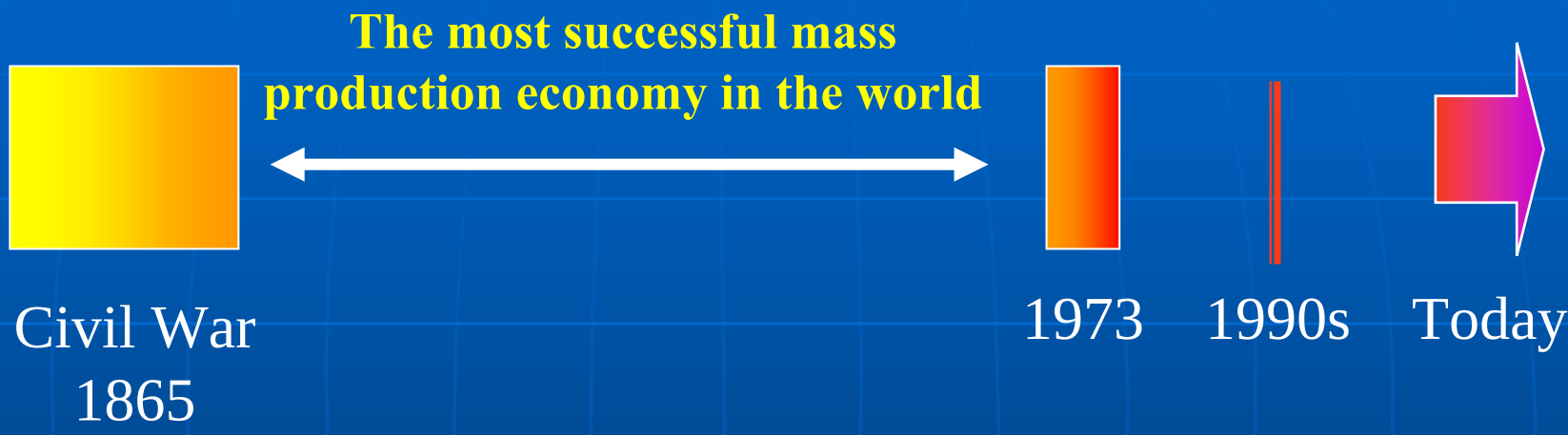
“The Second Half of the Chessboard”

Doubling Every 14-16 Months Will Continue Until 2020

Changes in Work and the Opportunities to Adjust

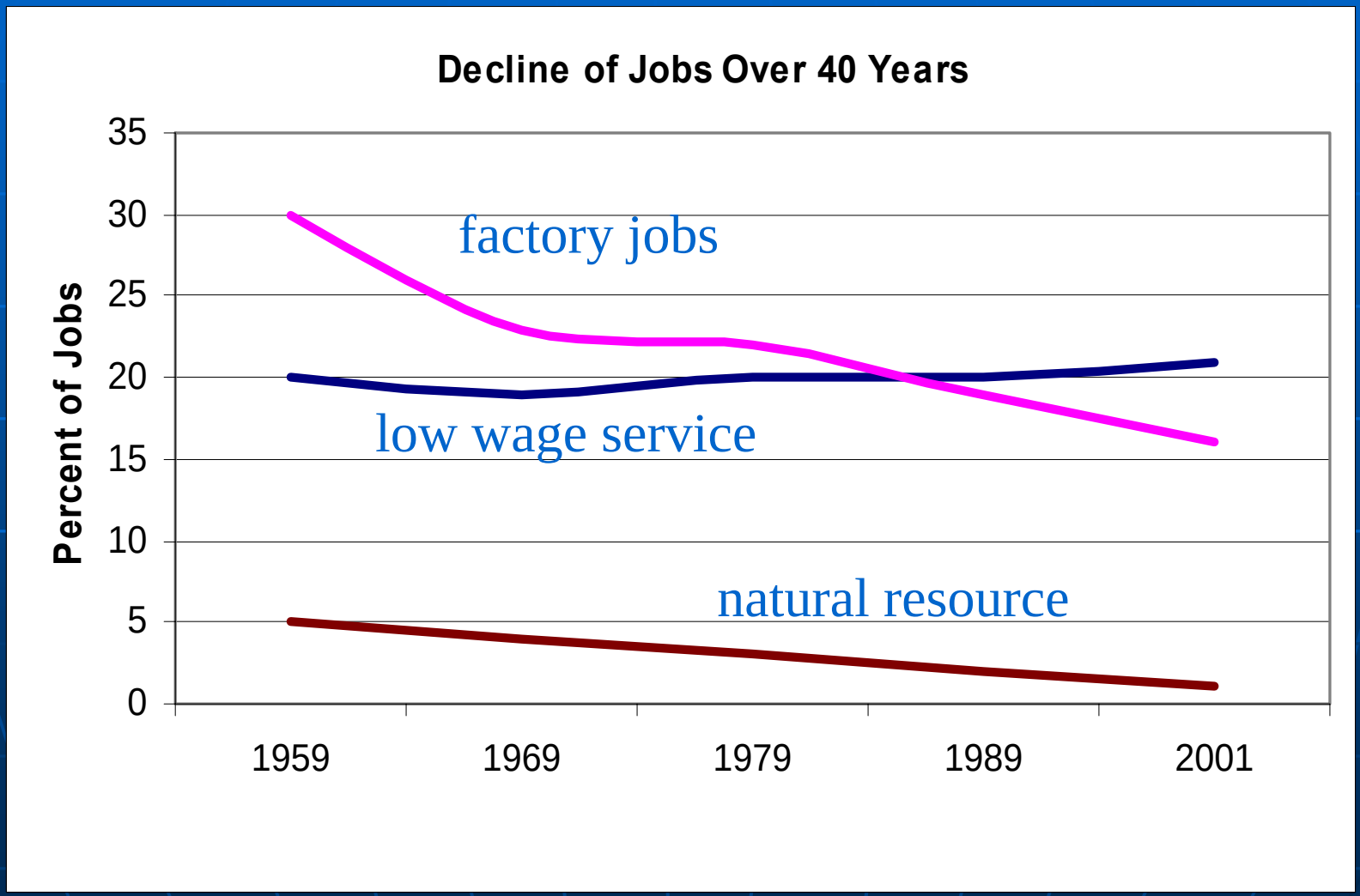


It Was a Good Run for the United States

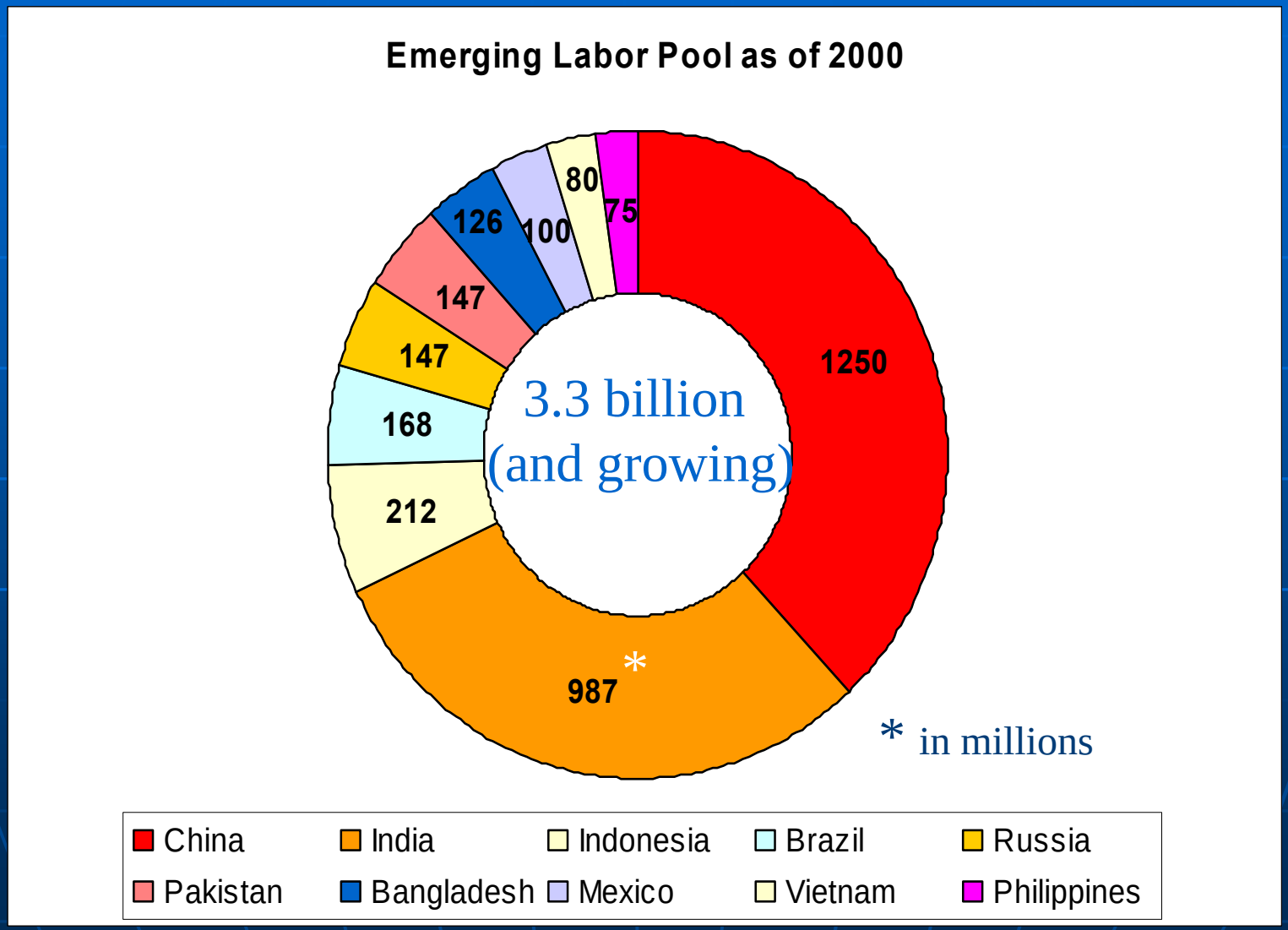


- 1946 to 1972
  - The “golden age” in the US: Family income doubled
  - Devastated European and Asian economies finally dig out of World War II, creating export economies with highly trained workers
- 1973 – 1990
  - Wage differences for high school and college grads
    - 40% higher for college grads in 1973
    - 90% higher for college grads in 1990

Areas of Decreasing Employment



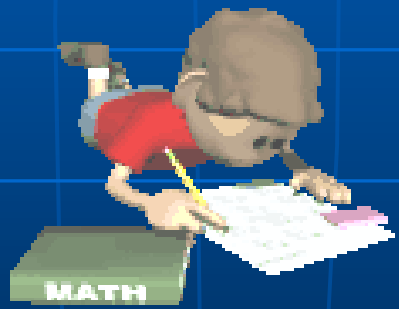
The Endless Supply of Low Skilled Labor



Source: Tonelson, A. (2002). ***The race to the bottom: Why a worldwide worker surplus and uncontrolled free trade and sinking American living standards.***

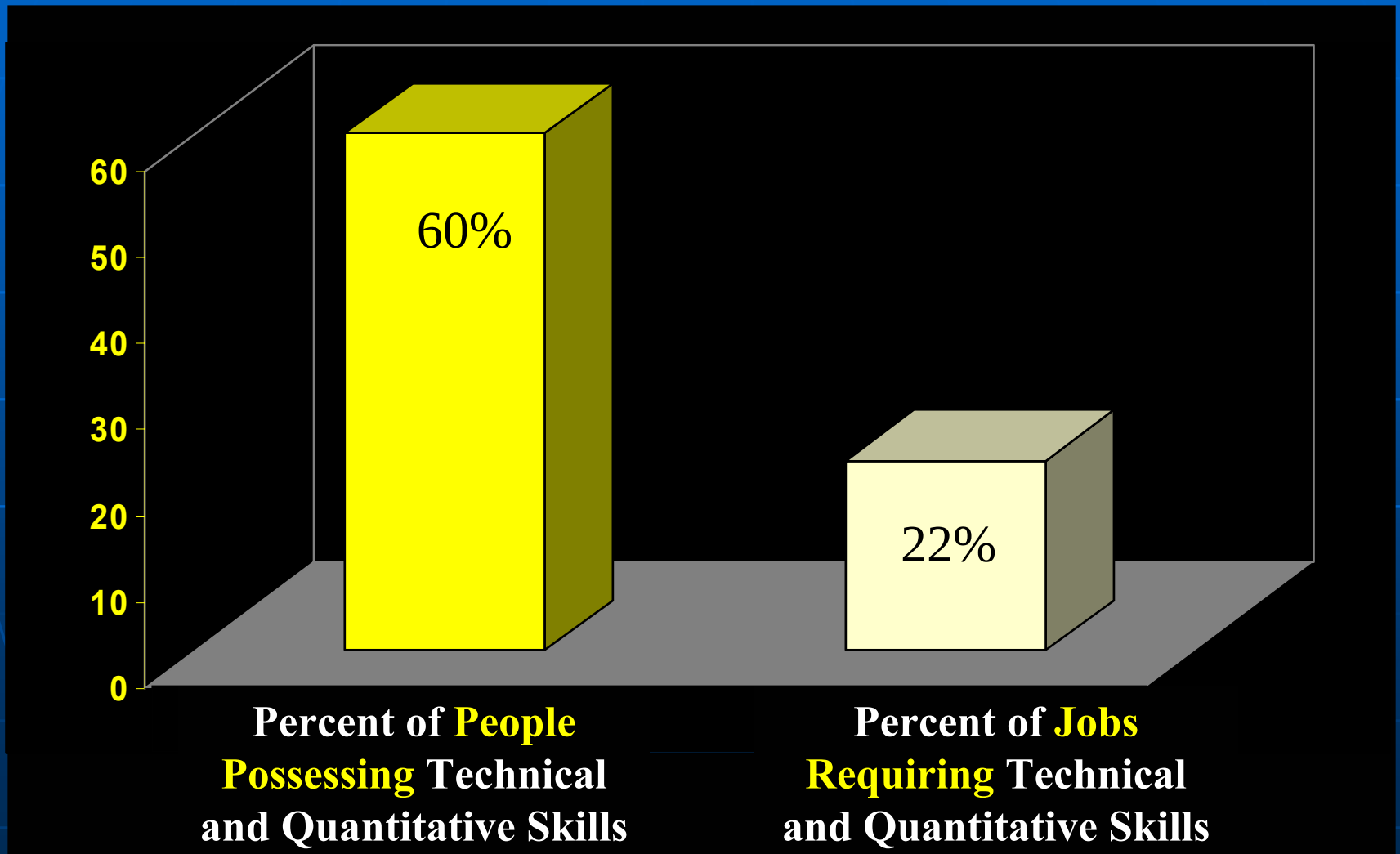
What All of This Means in the Near Term

A 7th grader in 2004 - 2005



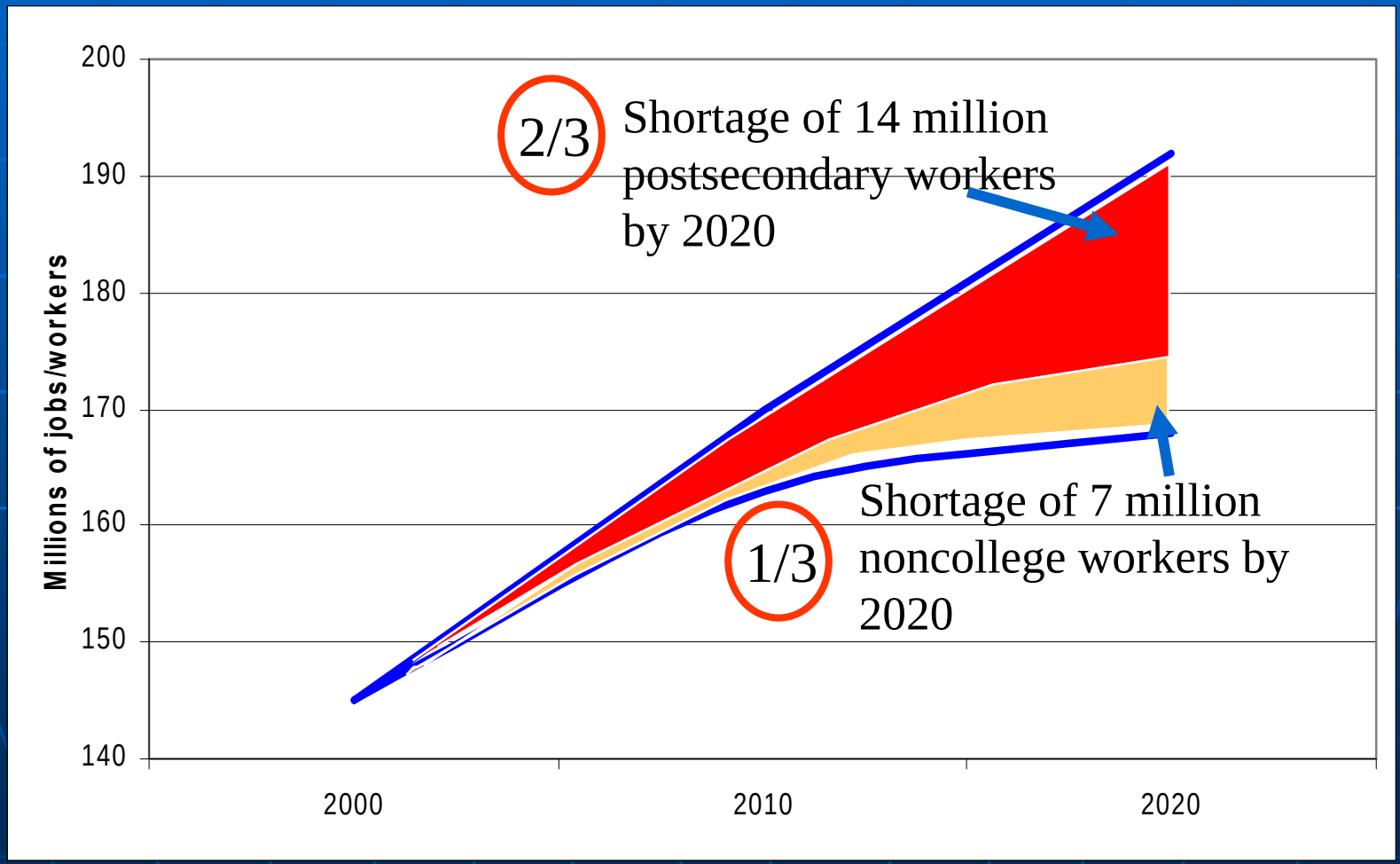
A high school graduate in 2010

Job Demands by 2010



US Dept of Labor

The Gap in Labor Force Growth and Job Growth through 2020



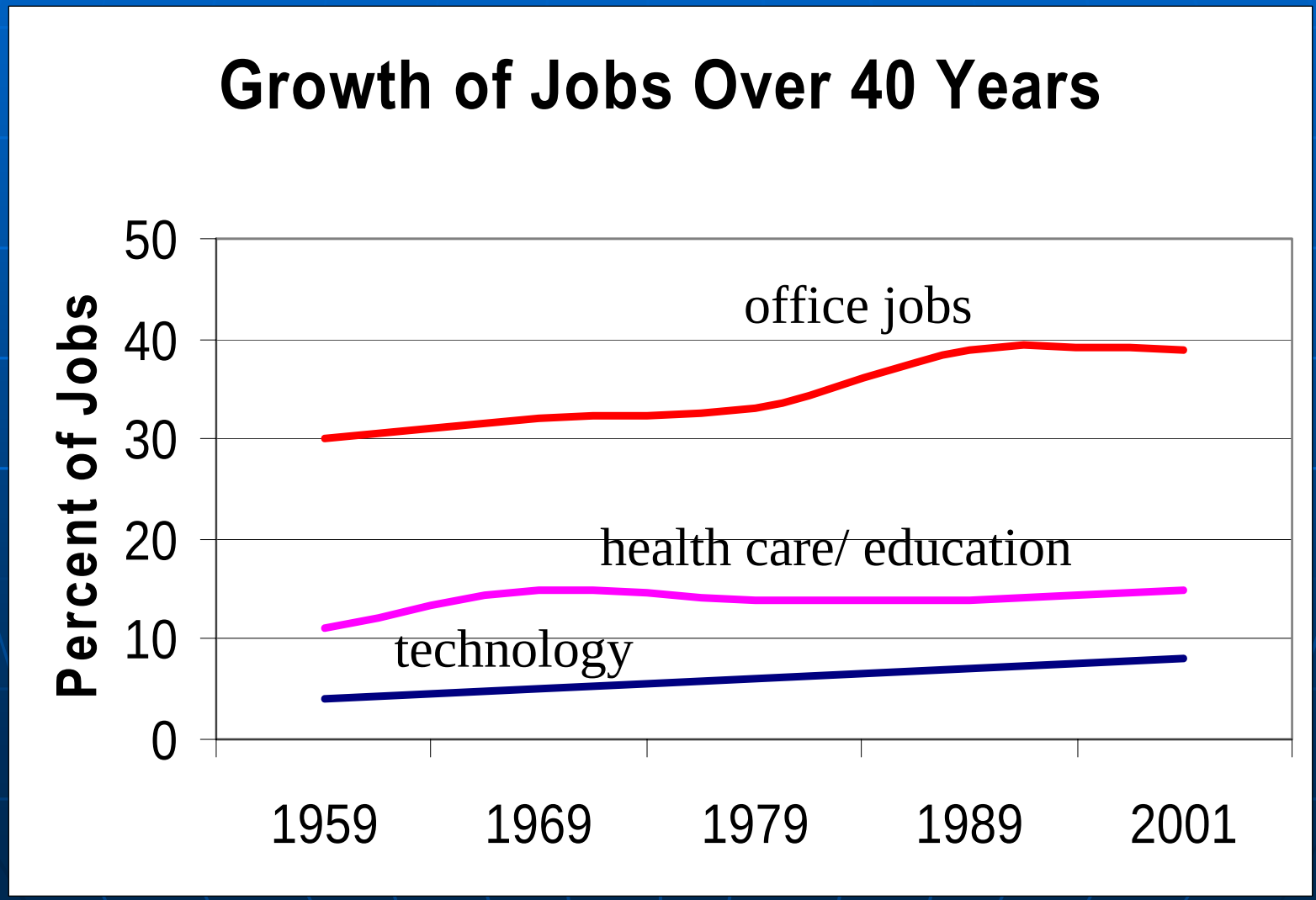
Carnevale and Desrochers, 2003

Rising Employment: The Office Economy

- Regardless of industry
  - Office workers are 41% of the workforce
  - They account of 50% of earnings
  - They are 65% of all managers and professionals

- Low skill service sector is 20% of the workforce
  - Relatively unchanged since 1959 but it will increase without continued increases in educational attainment

Areas of Increased Employment

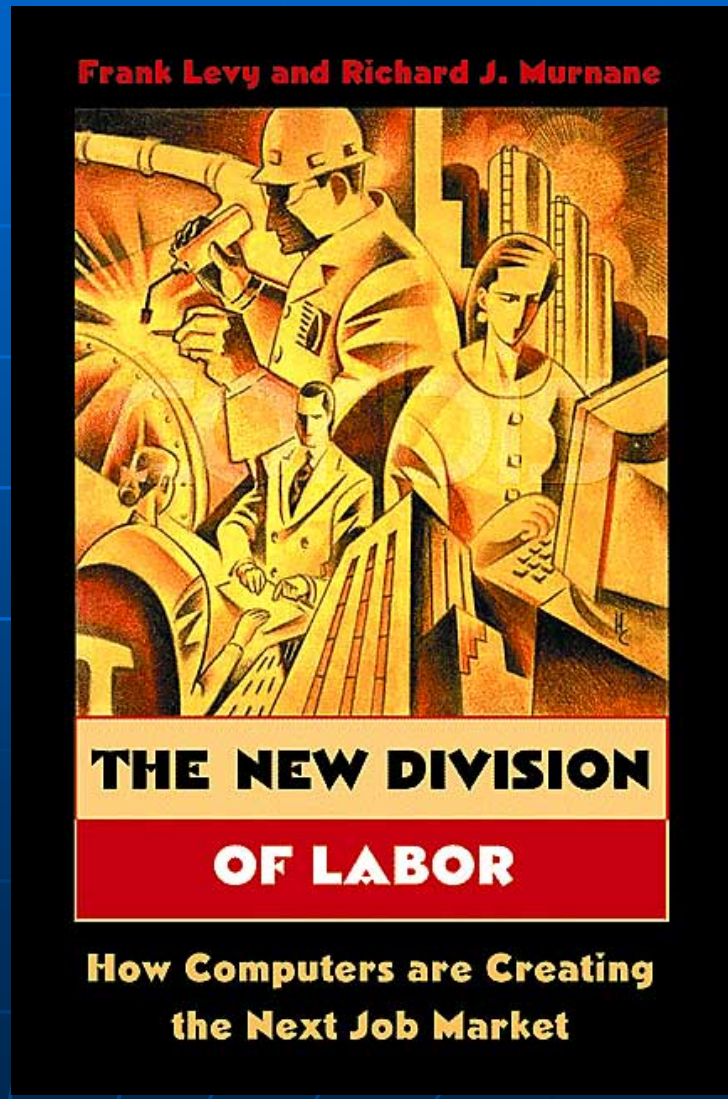


Office Work

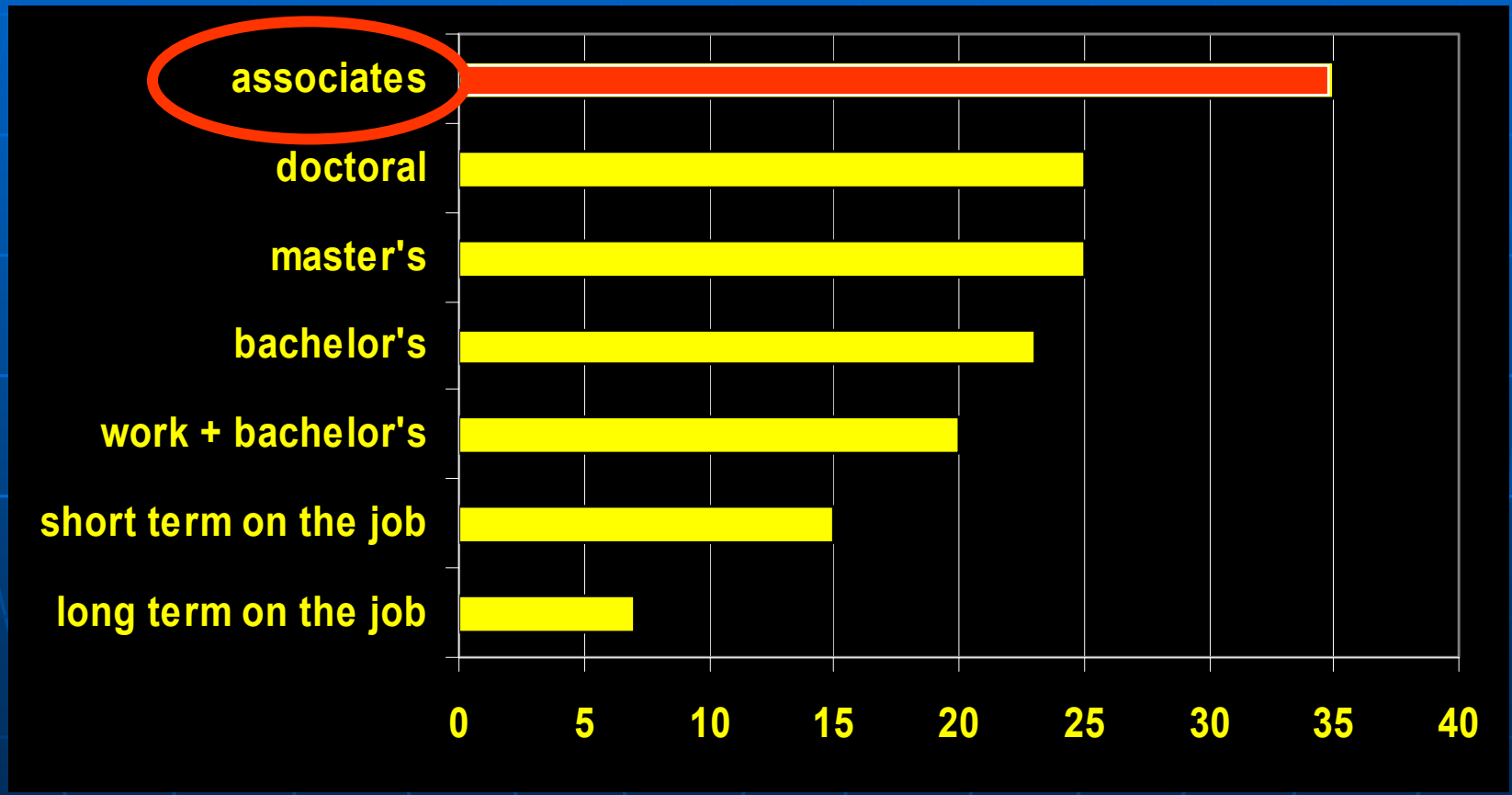
The race *against* technology

- Communication skills
- Problem solving
- Pattern analysis
- Ability to *work with* technology

-- *Levy & Murnane* (2004)



Percent Change in Jobs by Source of Education 2000-2010



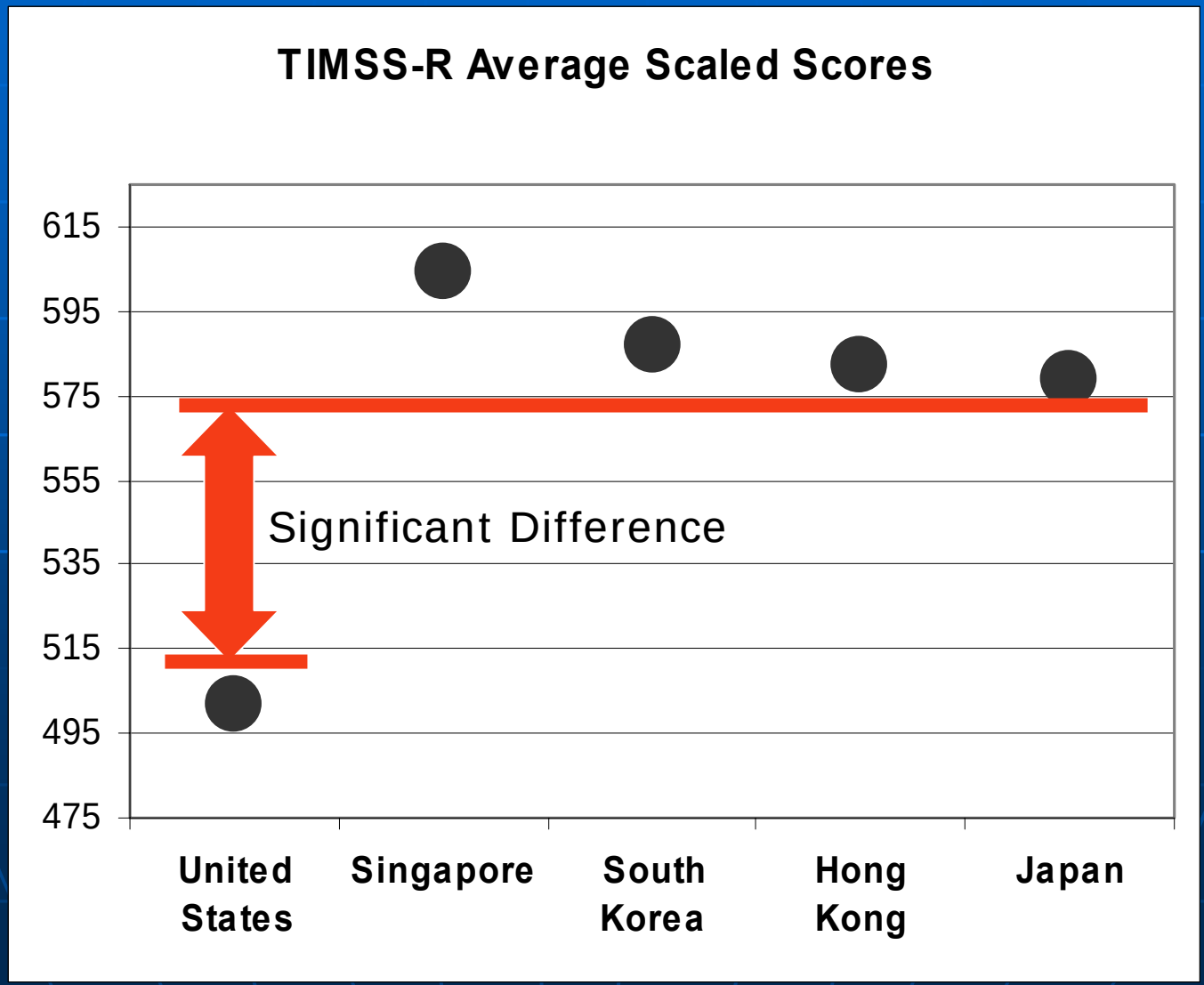
US Dept of Labor, 2003 Occupational Outlook

What Does All of This Mean for Mathematics Education and Students with Disabilities?

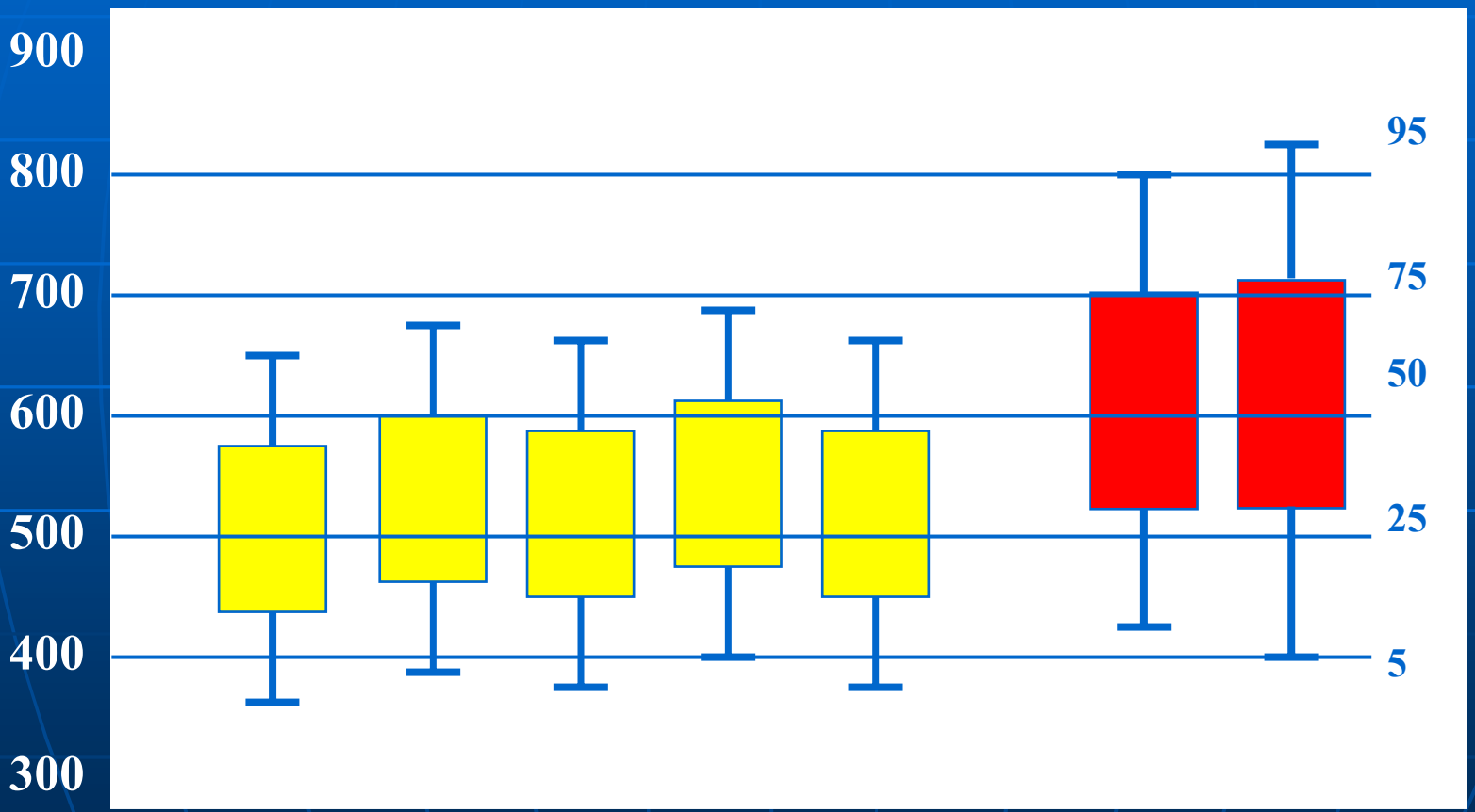
- In 25 years, we've gone from:
  - 1980s – back to basics
  - 1990s – the high standards movement
  - 2000s – high standards + accountability
- What is the *ALL* in ALL STUDENTS WILL ….?

Woodward, J. (2004). Mathematics reform in the US: Past to present. Journal of Learning Disabilities

The Question of ALL Students Internationally



Variability and the 8th Grade TIMSS



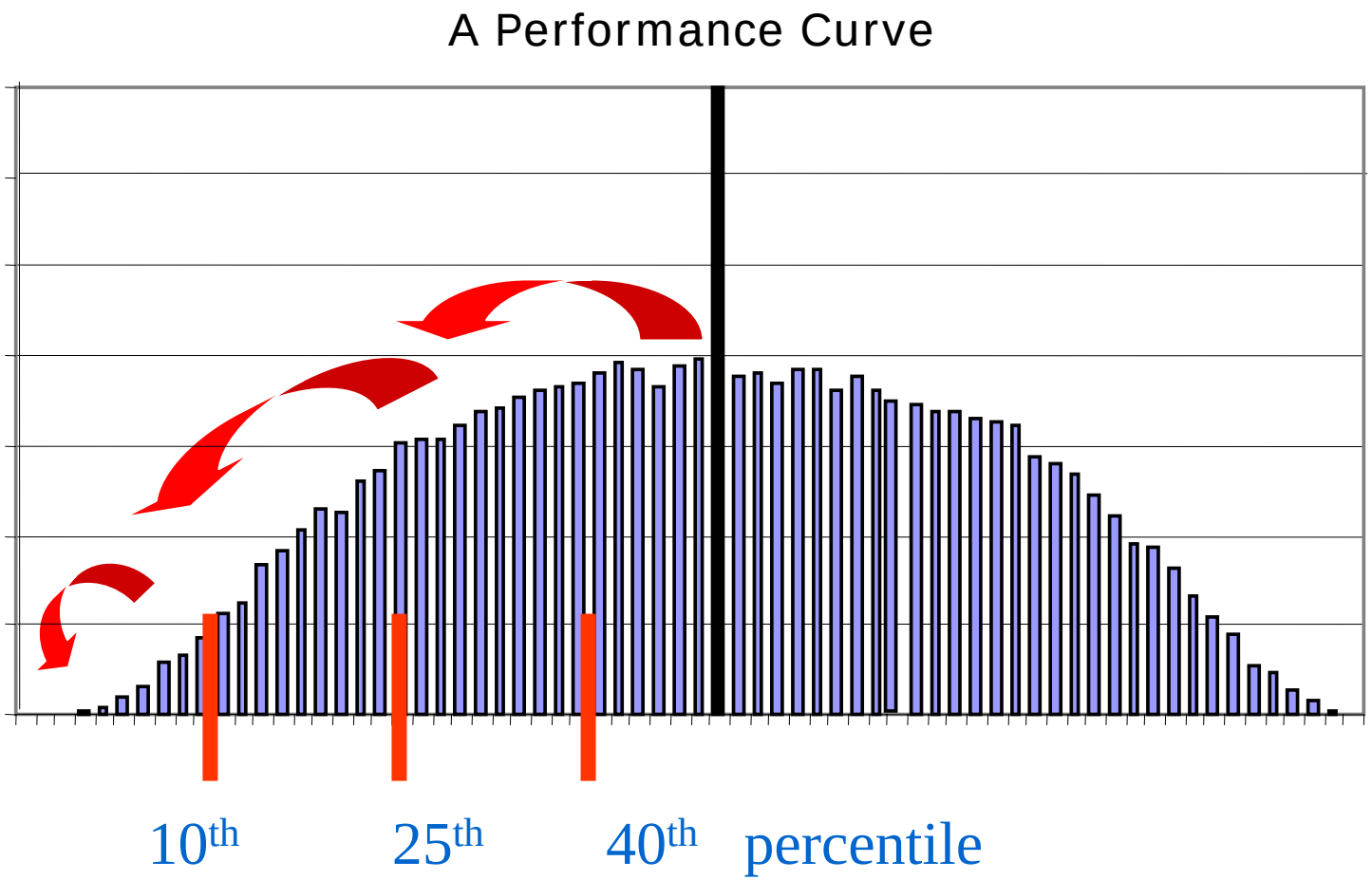
Linn, 2000

Now the Question of ALL Students in the United States Who Are

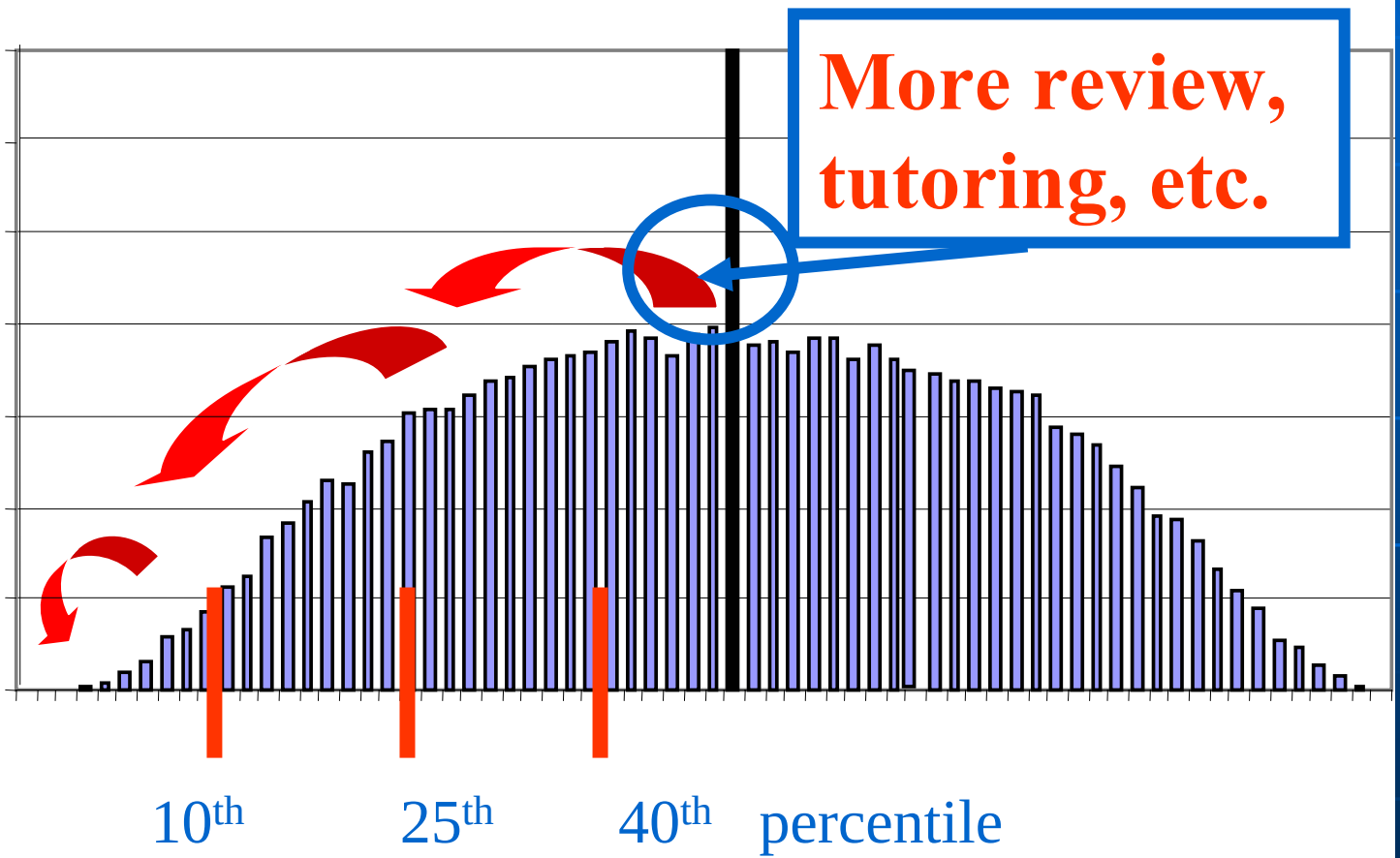
– by Some Measure –

Below the Academic Midpoint in Mathematics

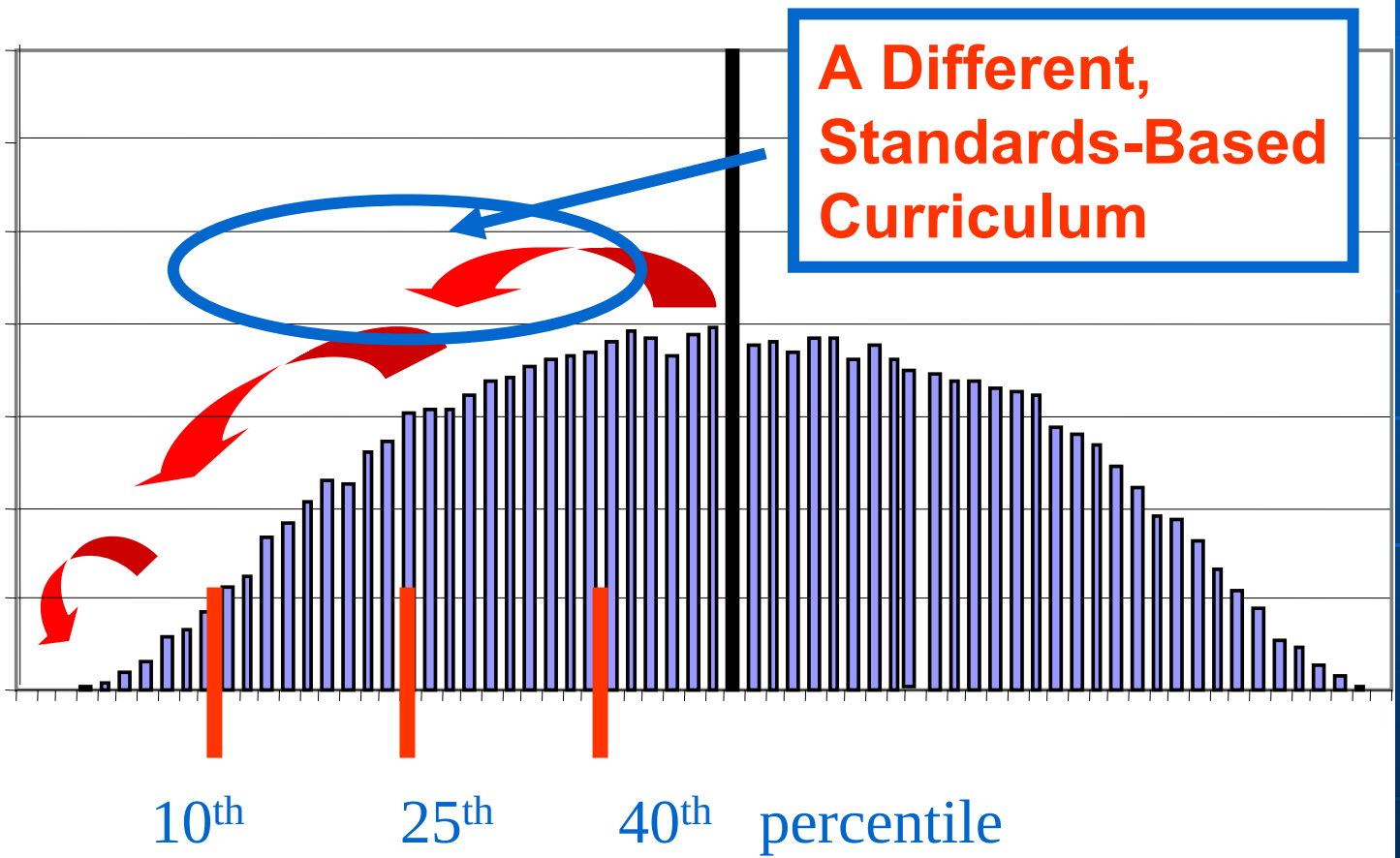
Students Below the Midpoint Require Different Solutions



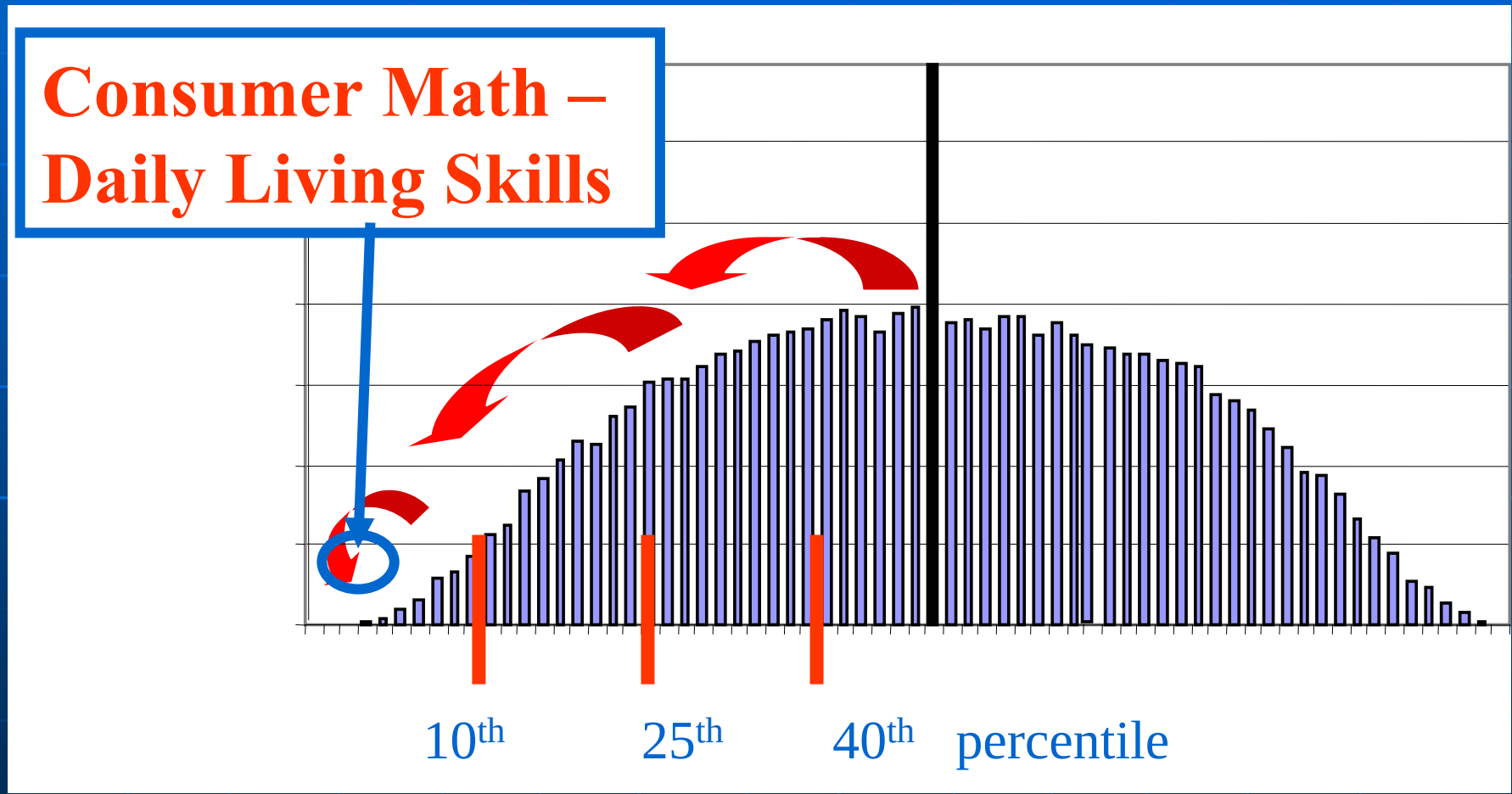
Just Below the Midpoint



The Low Achiever/ LD Range



Students with Significant Cognitive and Physical Difficulties



Implications for Special Education

- Many students with mild disabilities can benefit from new approaches to mathematics that move them in the direction of algebra

- Students with more severe disabilities have mathematical needs best articulated through IEPs

  - By definition, their goals should be *customized*
  - *By the nature of their disabilities,* moving toward algebra is neither feasible nor relevant

For Those Students with Disabilities a New Way of Thinking about Mathematics

- Early Intervention – Is There an Analogy with Reading Interventions?
- Are Best Instructional Practices for Math from the Past *Best*?
- Rethinking the Problem Means Doing More than Acquiring New Materials

Number Sense as Phonemic Awareness

As the national focus in education shifts from reading to math, some hope that there is a similar kind of early intervention in math, particularly with students at-risk for special education.

Put simply, “Are there 5 key principles in math like there are in reading?”

Math Develops Naturally *Before* School

Informal Development of Counting, Subitizing, Cardinal numbers, Ordering, Measuring

And how well a young child does “in math” is task dependent

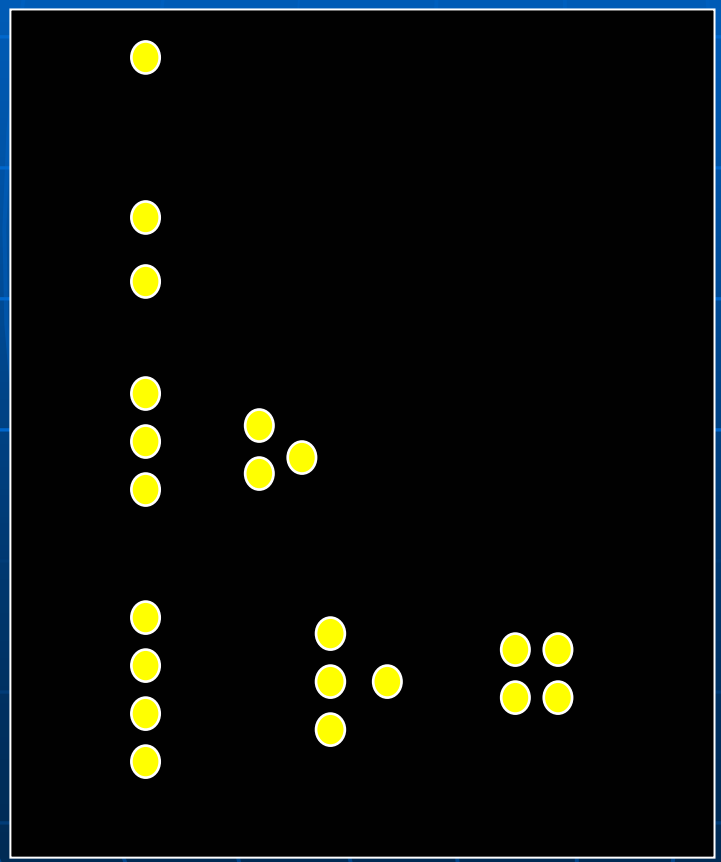
Intervening in the Early Years

This Would be a Major Mathematical Error

- When shown 4, the student
  - Says “4”
  - Counts to 4
  - Writes 4
- When shown • • • • the student
  - Says “4”
  - Counts to 4
  - Writes 4

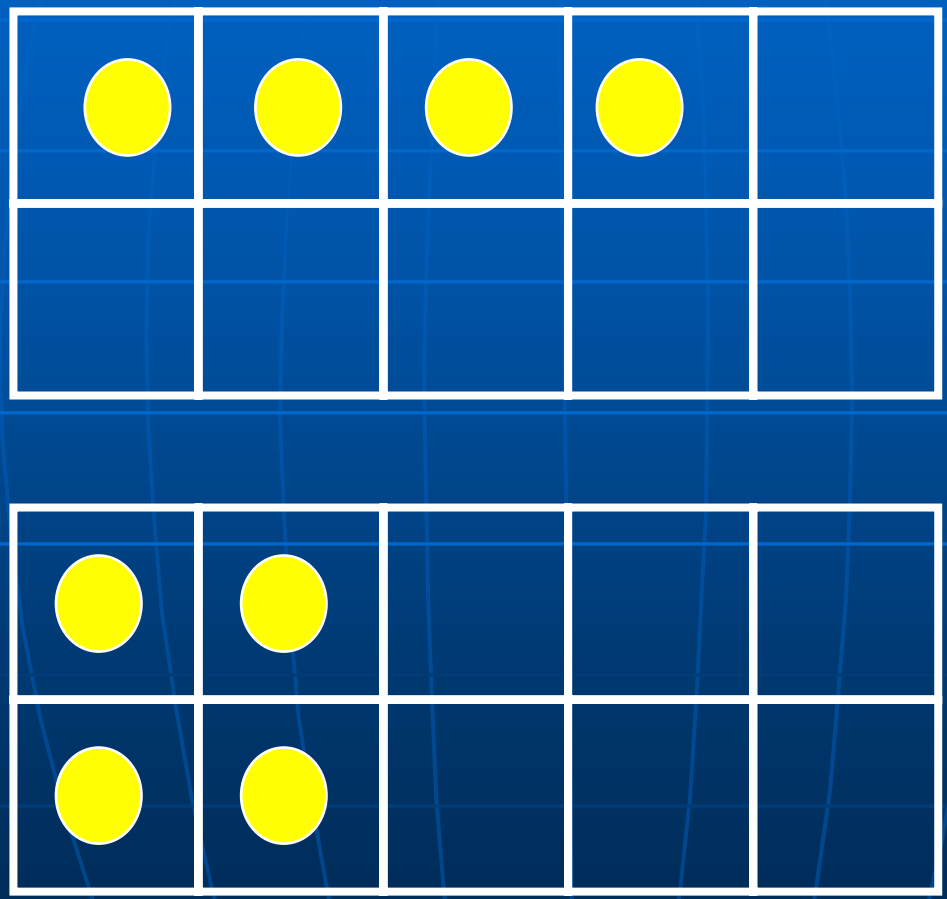
Counting and Identifying Numbers

4 is “four” but It Is Also



Patterns

4 Is Also



Near 5

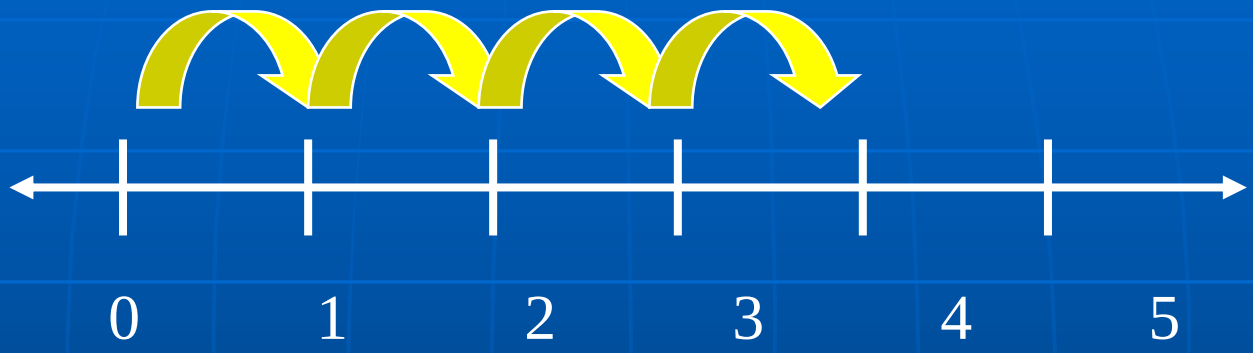
5 – 1

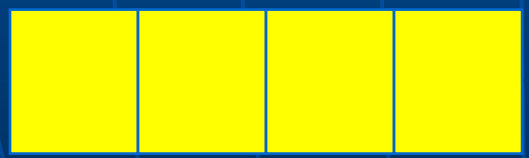
10 – 6

4 + 6 = 10

“Doubles”

4 Is Also





An Array

4 Is Also

$4 = 4$

$4 = 4 + 0$

$4 = 3 + 1$

$4 = 2 + 2$

$4 = 2 \times 2$

$4 = 2^2$

$4 = \sqrt{16}$

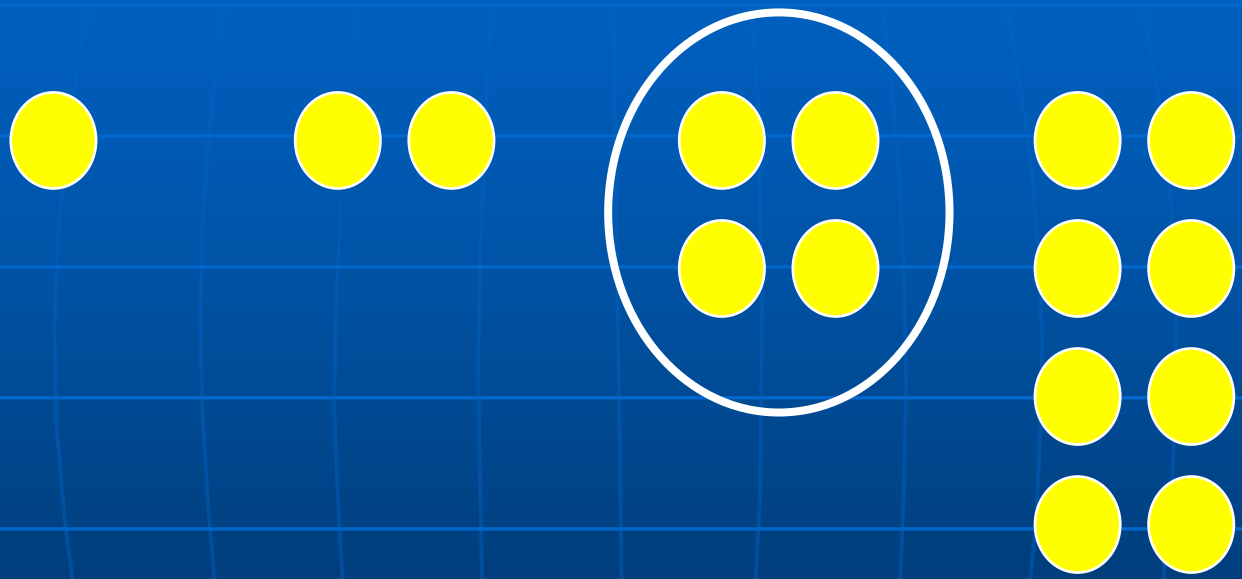
$4 = 8 \div 2$

$4 = 10 - 6$

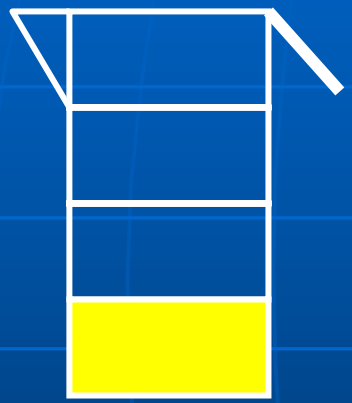
$4 = 4 + (4 \times 0)$

etc.

4 Is Also



4 Is Also



$\frac{1}{4}$

ssn# 349-77-6549

427-9436

1244 W. 10th Place

first second third fourth

4 o’clock

4 Isn't A Number "By Itself"
(It's More Complex Than You Think)

$$4 + 4 \neq 8$$

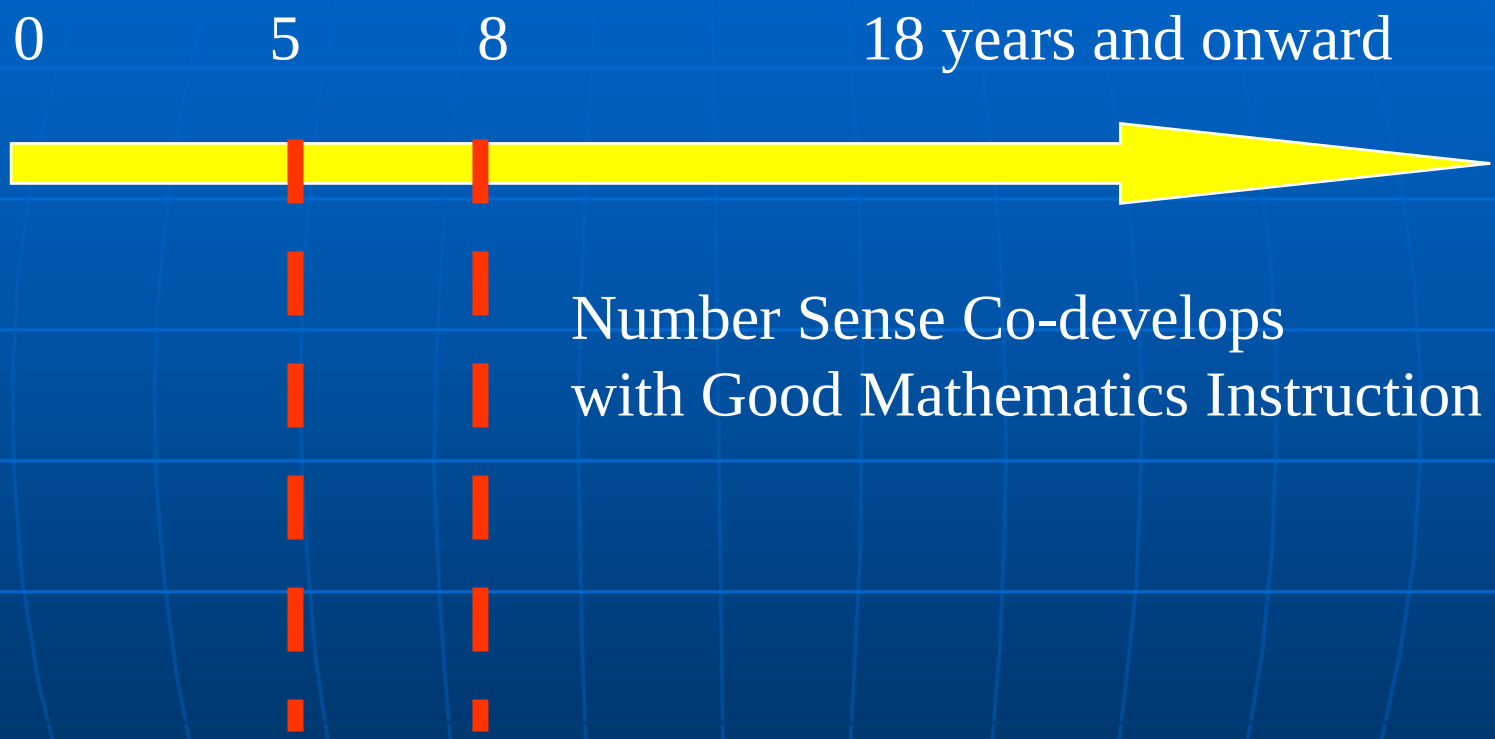
This Makes Cents When You Think About It

4 dollars + 4 dimes = $4.40

And There's More Than Counting and Identifying Numbers

- *The Content & Process Strands of the NCTM Standards*
  - Measurement
  - Geometry
  - Fractional Numbers (Pattern Blocks)
  - Collecting and Analyzing Data (Pictographs)
  - Pattern Analysis
  - Reasoning, Communicating, Representing
  - Problem Solving

Number Sense is Also



How Good is Your Number Sense?

$\frac{4}{7} \overset{?}{<>} \frac{2}{5}$

1 2 3 4 5 6 7 9

x 9

1 1 1,1 1 1,1 1 1

why?

20x + 140x + 13x + 70x + 7x – 20x =

15, 987, 588

- 1,999

Are Best Practices from the Past *Best*?

- Math research was conducted by generalists
- Math research comes from the 1980s Back-to-Basics era
  - Instruction stressed rote learning
  - It never envisioned the math literacy (or the technology) students need for today or the future
- The students used in the research are highly variable
  - As mentioned earlier, we need different solutions for different students who are below the 50th %ile

The Problem with Past Best Practices

Some Examples

- Key Instructional Principles
  - Teach computational skills sequentially and/or hierarchically to ***mastery***
  - Insure high levels of success at each step
  - Provide practice to the point of overlearning
  - Teach students key words for solving problems

Teaching Facts: Lots and Lots of Drill and Practice

(How Else Would You Do It?)

12	12	12	12	12	12	12
- 3	- 4	- 5	- 6	- 7	- 8	- 9

13	13	13	13	13	13
- 4	- 5	- 6	- 7	- 8	- 9

14	14	14	14	14
- 5	- 6	- 7	- 8	- 9

But Given the Memory Demands, We Get:

$$\begin{array}{r} 12 \\ -\ 7 \\ \hline 6 \end{array}$$

(or 4)

$$\begin{array}{r} 12 \\ -\ 7 \\ \hline \end{array}$$

11
10
9
8
7
6
5

$$\begin{array}{r} 12 \\ -\ 7 \\ \hline ? \end{array}$$

COMPUTATIONS: A Hierarchy to Skill Development

5	35	357	357
x 3	x 3	x 3	x 43

It Looks Like Multiplication

$$
\begin{array}{r}
^{2}\cancel{1}\ ^{2}\cancel{2} \\
357 \\
\times\ 43 \\
\hline
1071 \\
+\ 1428 \\
\hline
15251
\end{array}
$$

30 ***procedural*** steps

The Direct Translation Method

Mattie loved to read books, so every day Mattie read 3 books. Mattie read 18 books in all. How many days did Mattie read books?

- Look for key words *each* or *every*
  - *Each* or *every* mean to multiply or divide
- Look for the big number – it tells how many there are in all.
  - This problem tells how many books Mattie read in all – that's the big number.
  - The remaining "2 small numbers" are part of a family
  - In this case, we divide 18/3 = 6
- Go on to the next problem

Darch et al. (1984) Explicit Instruction in Mathematics Problem Solving. Journal of Educational Research

Another Look at Direct Translation

Problem 1

Kara has 34 coins, and she gets 12 more coins from her piggy bank. How many coins does she have?

Problem 2

Lindsey has 57 tennis balls. If Lindsey gets 16 more, she will have as many as Morgan. How many tennis balls does Morgan have?

Problem 3

Erin has 46 comic books. He has 15 more comic books than Jason has. How many comic books does Jason have?

Woodward, J., Baxter, J., & Scheel, C. (1997). It's what you take for granted when you take nothing for granted: The problems with general principles of instructional design. In T. Scruggs & M. Mastropieri (Eds.), Advances in Learning and Behavioral Disorders (Vol. 11), (pp. 199-234). New York: JAI Press.

So What Can We Do?

- Keys Elements from Special Education Research
  - Develop Strategic Knowledge
  - Work on Task Persistence
  - Provide Distributed Practice
- Math Reform and Cognitive Psychology
  - Visual Representations and Classroom Talk
- Findings from International Comparative Research
  - Conceptual Understanding as a *Habit of Mind*
  - Fewer Topics in Greater Depth

Examples of Number Sense

Math Facts Are Still Important

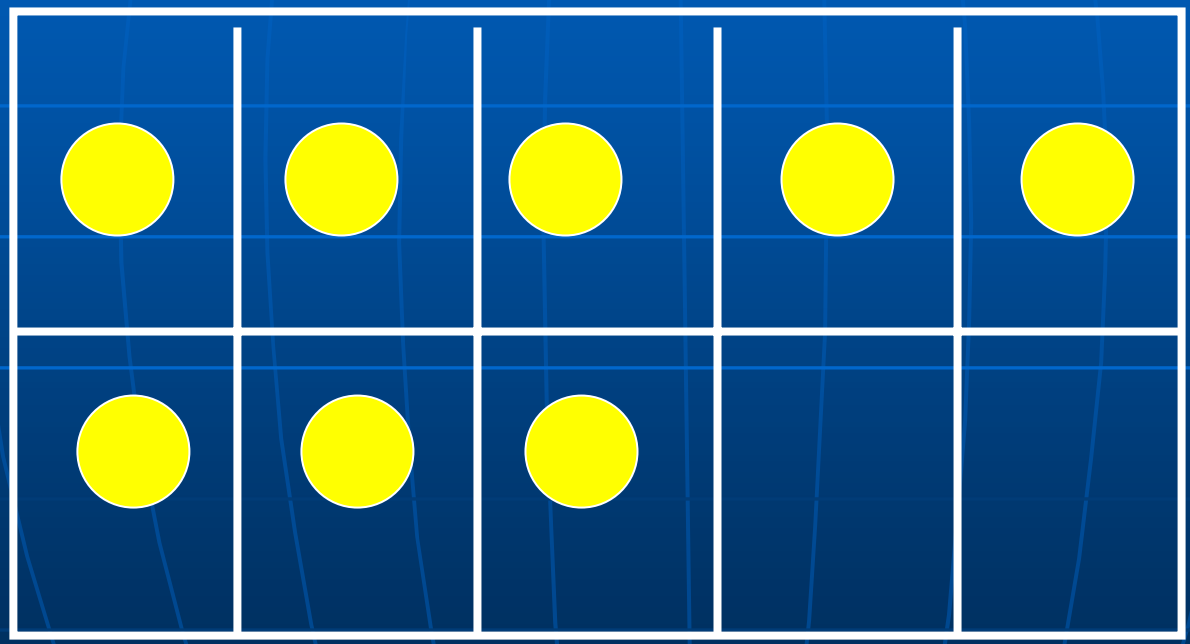
- Teach Strategies
- Use Visual Representations
- Have Timed Practice for Automaticity
- Link Facts to Approximations

Fact Strategies

- Addition
  - Doubles (3 + 5 = 4 + 4)
  - Doubles Plus 1 (4 + 5 = 4 + 4 + 1)
  - Through 10 (8 + 5 = 8 + 2 + 3)

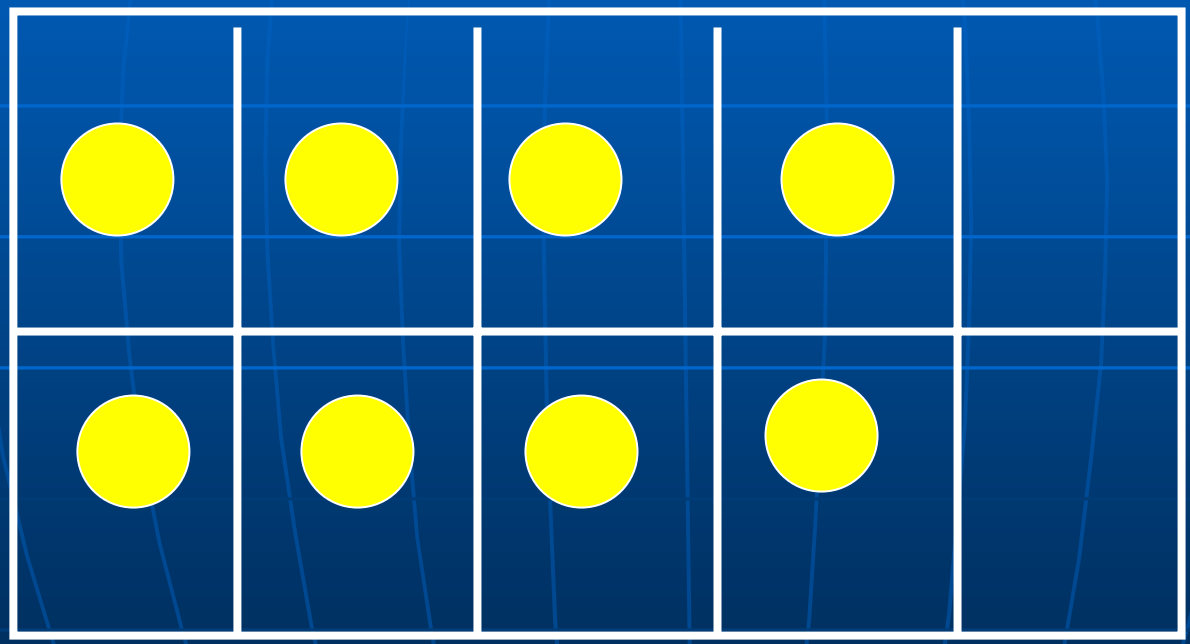
Visual Representations for Facts: Ten Frames

5 + 3



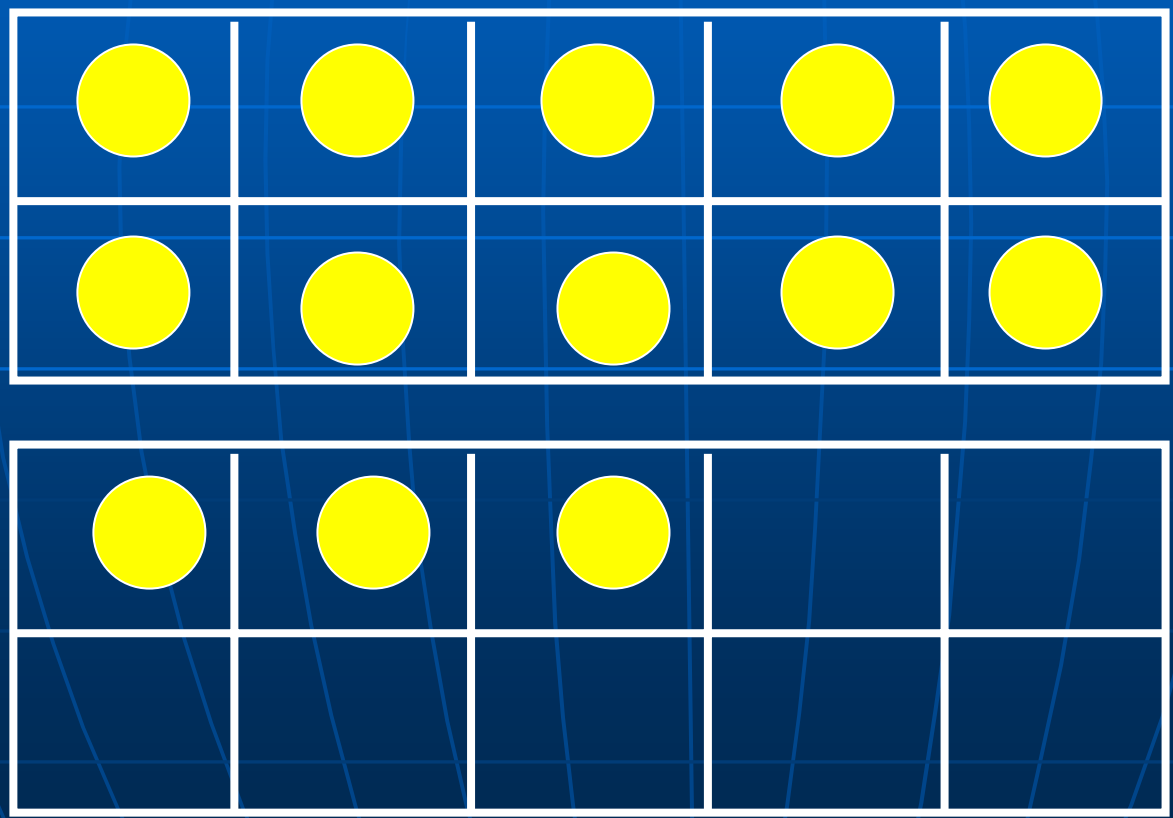
Ten Frames

5 + 3 as *doubles (4 + 4)*



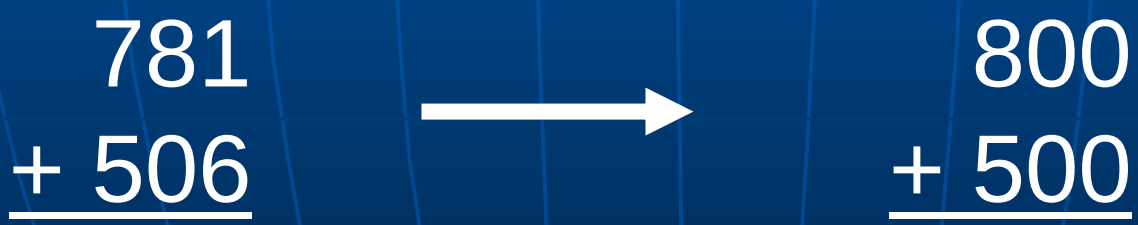
Ten Frames

8 + 5 = 8 + 2 + 3 *(thru 10s)*



Facts and Number Sense

extended facts



approximations

Examples of Number Sense

Controlling Procedural Knowledge and Stressing the Concept of Place Value

Regrouping

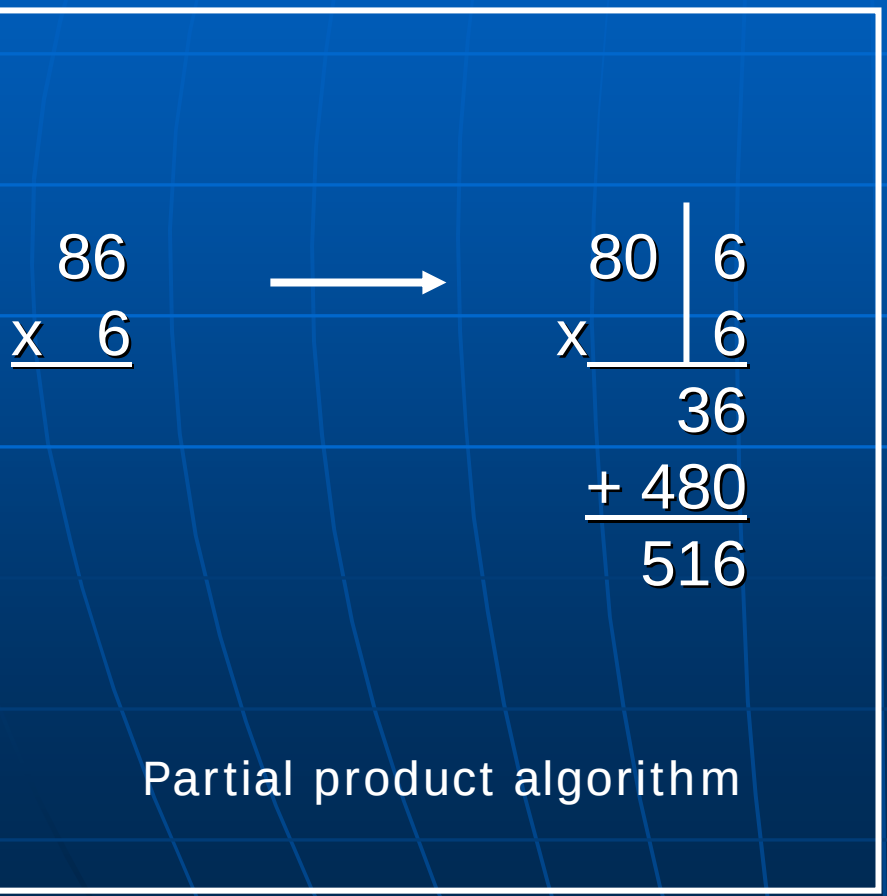
Informal commutation

37 + 42 = 30 + 7 + 40 + 2

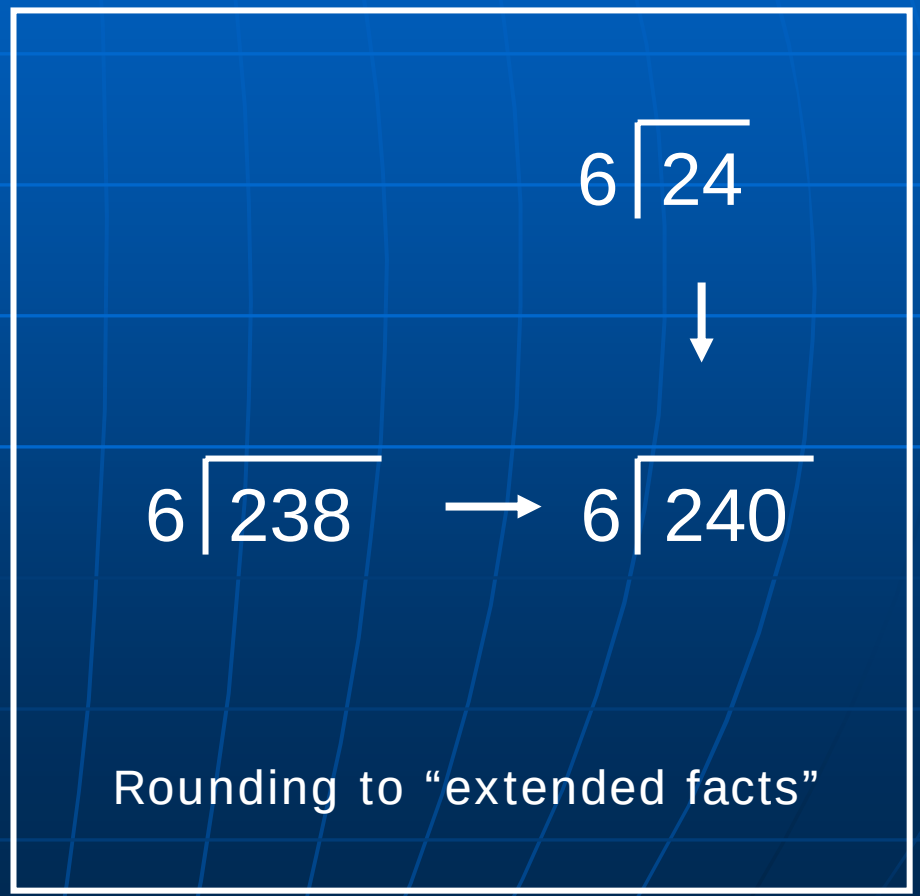
= 30 + 40 + 7 + 2

Examples of Number Sense

Controlling Procedural Knowledge and Stressing the Concept of Place Value



Partial product algorithm

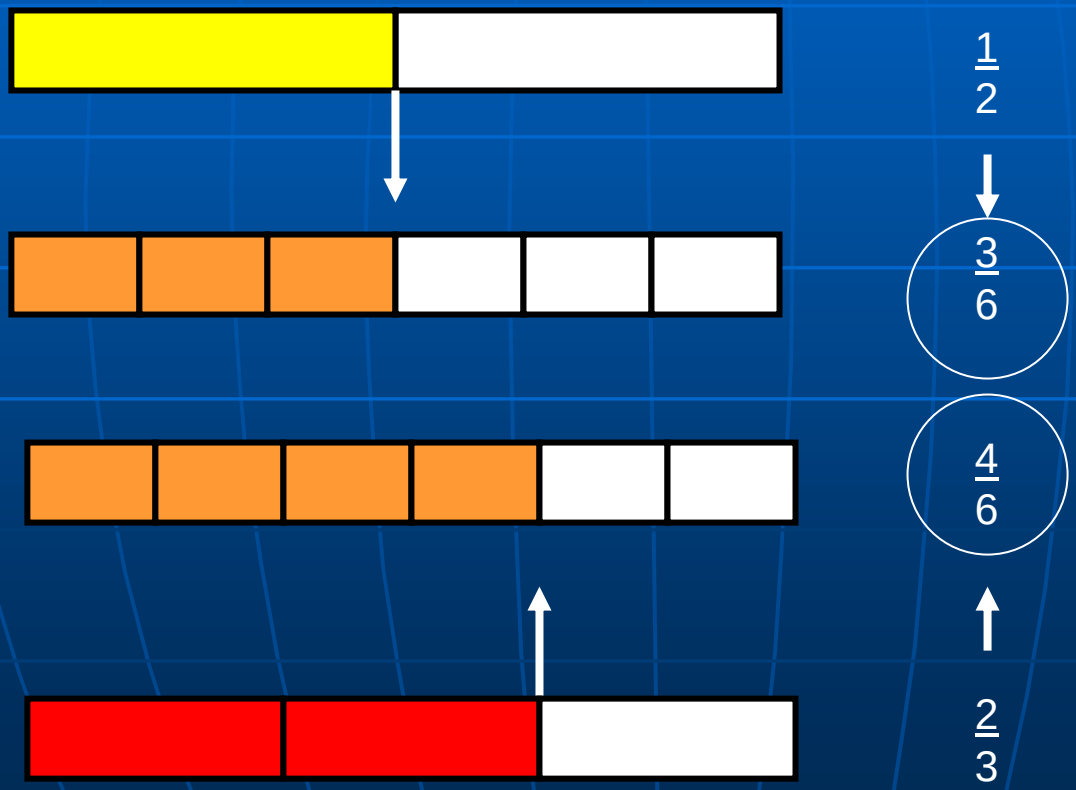


Rounding to “extended facts”

Moving Beyond Whole Numbers

Visual Representations and Rational Numbers

Equivalent Fractions



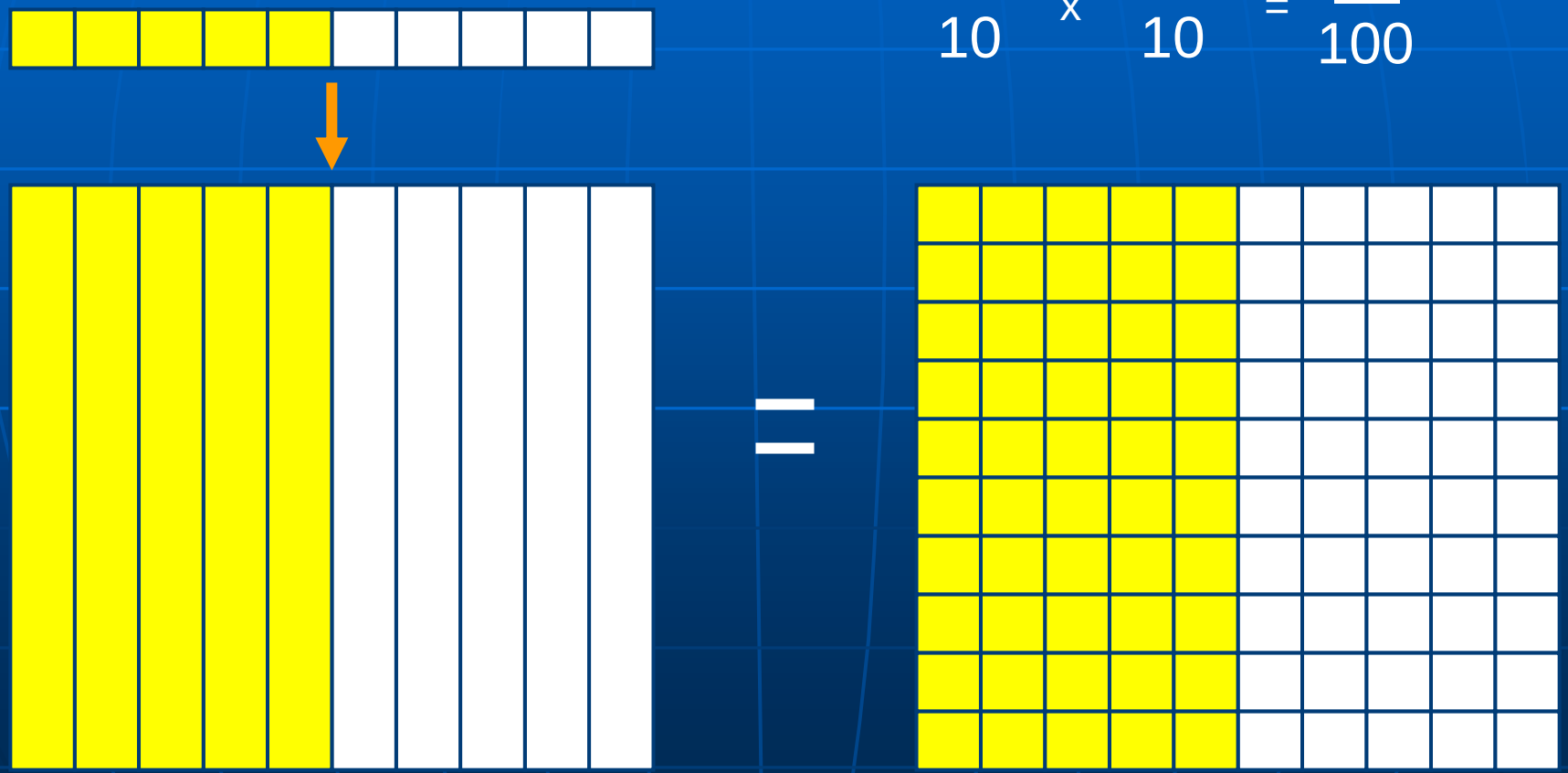
$$\frac{3}{6} + \frac{4}{6}$$

Moving Beyond Whole Numbers

Visual Representations and Rational Numbers

Fraction Bars to 100 Square Grids

$$\frac{5}{10} \times \frac{10}{10} = \frac{50}{100}$$



Decimal Numbers

Procedures vs. Number Sense

Compute

$$\begin{array}{r} 3.12 \\ \times\ 5.05 \\ \hline \end{array}$$

OR

Where does the decimal go?

3.12 x 5.05	=	1 5 7 5 6
3.12 x 50.5	=	1 5 7 5 6
.312 x 5.05	=	1 5 7 5 6

Problem Solving

Lists of these kinds do not develop problem solving

Mattie loved to read books, so every day Mattie read 3 books. Mattie read 18 books in all. How many days did Mattie read books?

Mattie loved to eat grasshoppers, so every day Mattie ate 3 grasshoppers. Mattie read 18 grasshoppers in all. How many days did Mattie eat grasshoppers?

Mattie loved to milk cows, so every day Mattie milked 3 cows. Mattie milked 18 cows in all. How many days did Mattie milk cows?

Make a Graph for Each Set of Data

Be Sure to Choose the Best Graph for the Data

The Hornets are a fast pitch softball team. Over the last 4 years, they have recruited players from all over the city to be on their team. As a result, they have won more games every year. Here is their record over the last 5 years.

Year	Won	Lost
1999	14	16
2000	17	13
2001	18	12
2002	21	9
2003	25	5

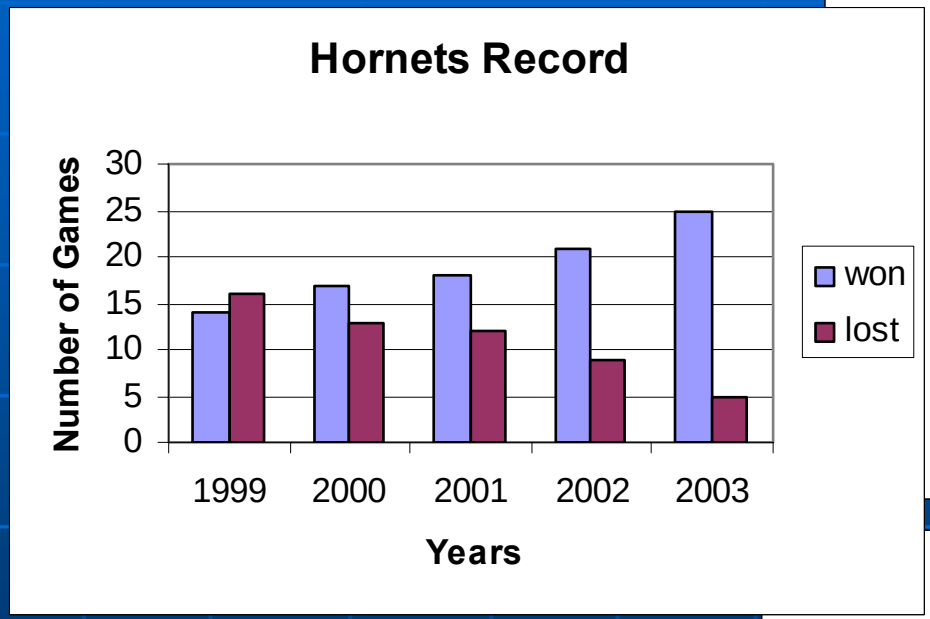
The Hornets are part of a Western Region League that has 120 teams at their level. Here are where the teams are located and how many teams in each region.

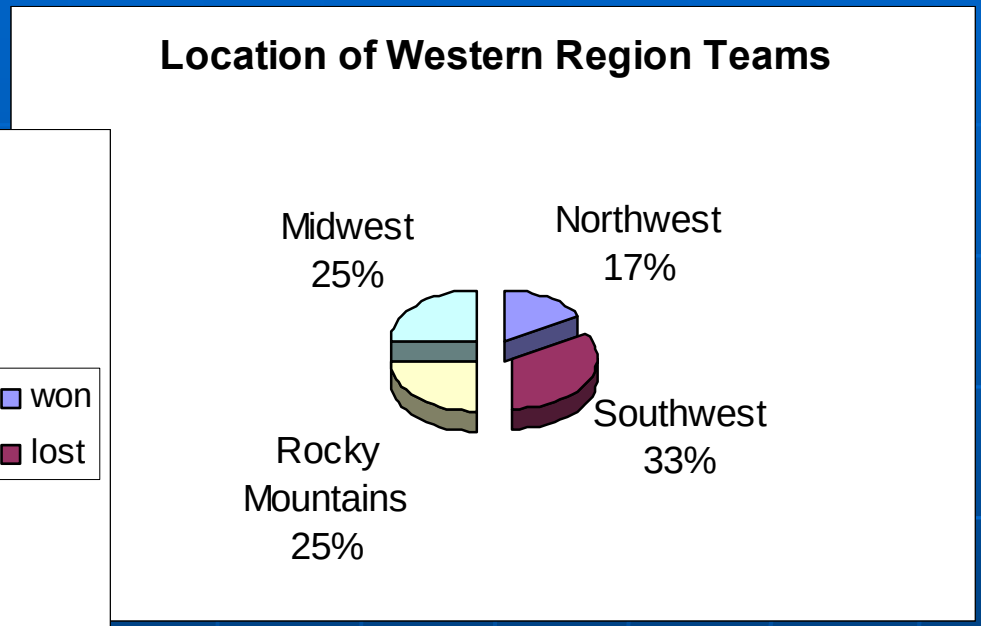
Region of the Country	Number of Teams
Northwest	20
Southwest	40
Rocky Mountains	30
Midwest	30

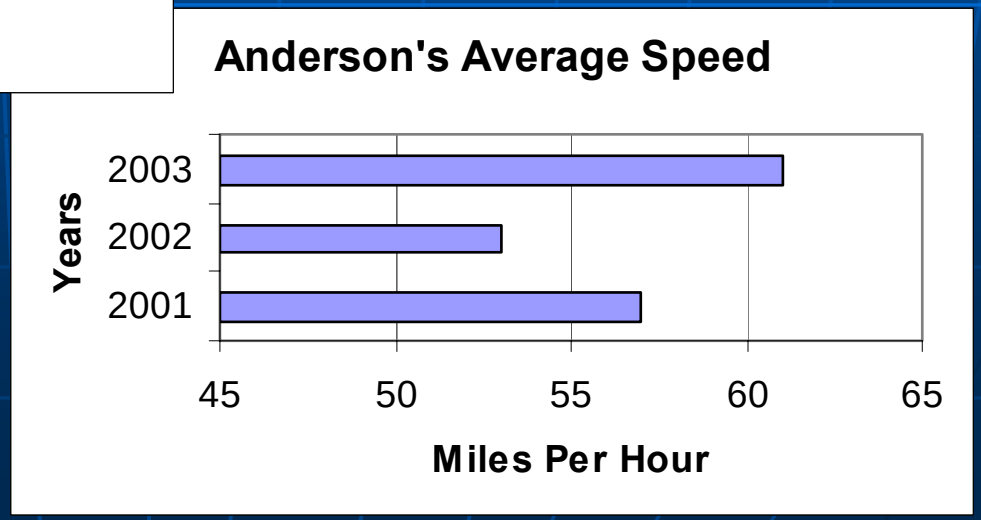
Kevin Anderson has been on the team as the starting pitcher since 2001. He has worked hard on his pitching. Here is the data on the average speed of his pitches for the last 3 years.

Year	Average Speed
2001	57 mph
2002	53 mph
2003	61 mph

Different Representations of the Data



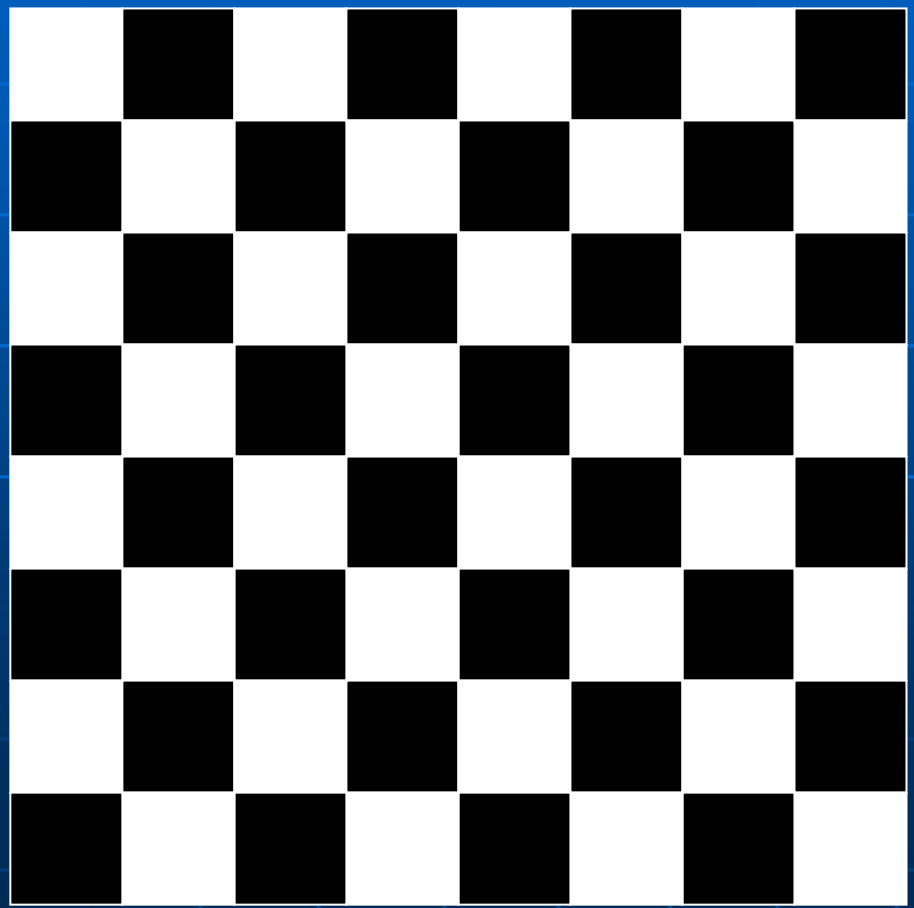




Data Analysis as Problem Solving

Good Problems Don’t Need Many Words

How many squares do you see? (Hint: the answer is NOT 64 or 65)



Problem Solving Strategies:

Look for patterns

Represent patterns using a table

A Sample of Statewide Assessments

A cereal manufacturer puts a 50¢ coupon in every eighth box and a free coupon in every hundredth box.

Tell how often a box will contain **both** coupons. Explain or show how you found your answer.

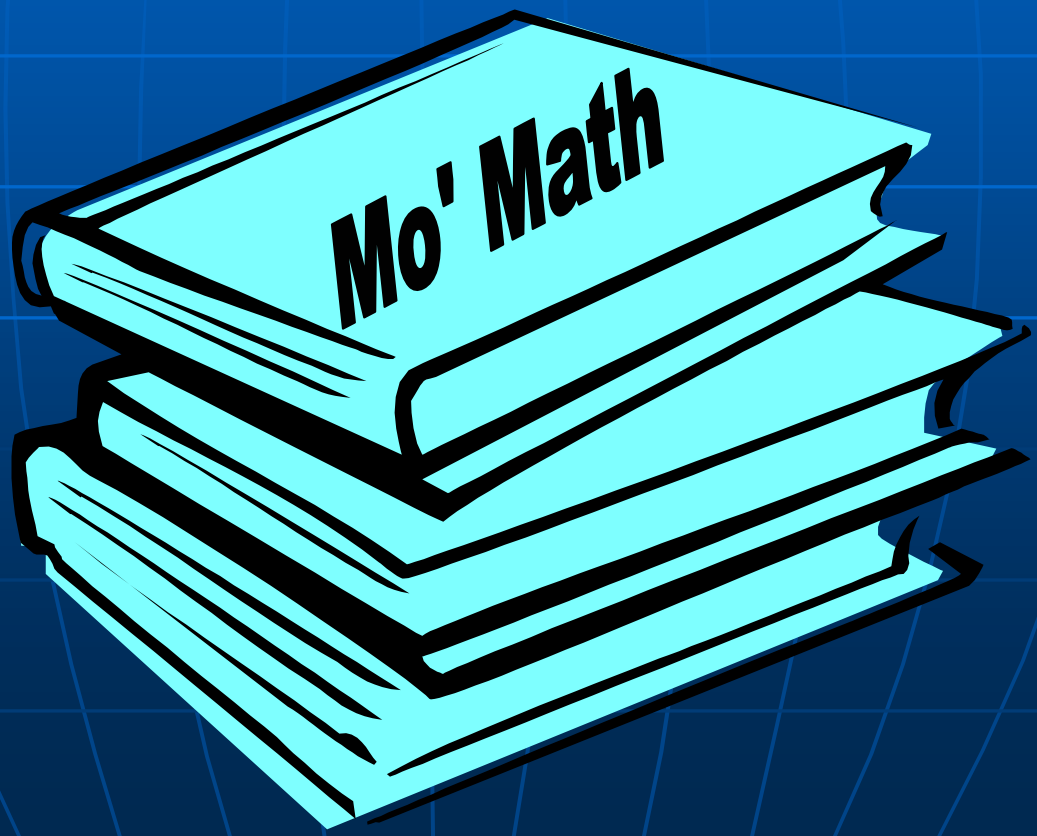
Every _____th box will contain both coupons.

(space for a written explanation telling why)

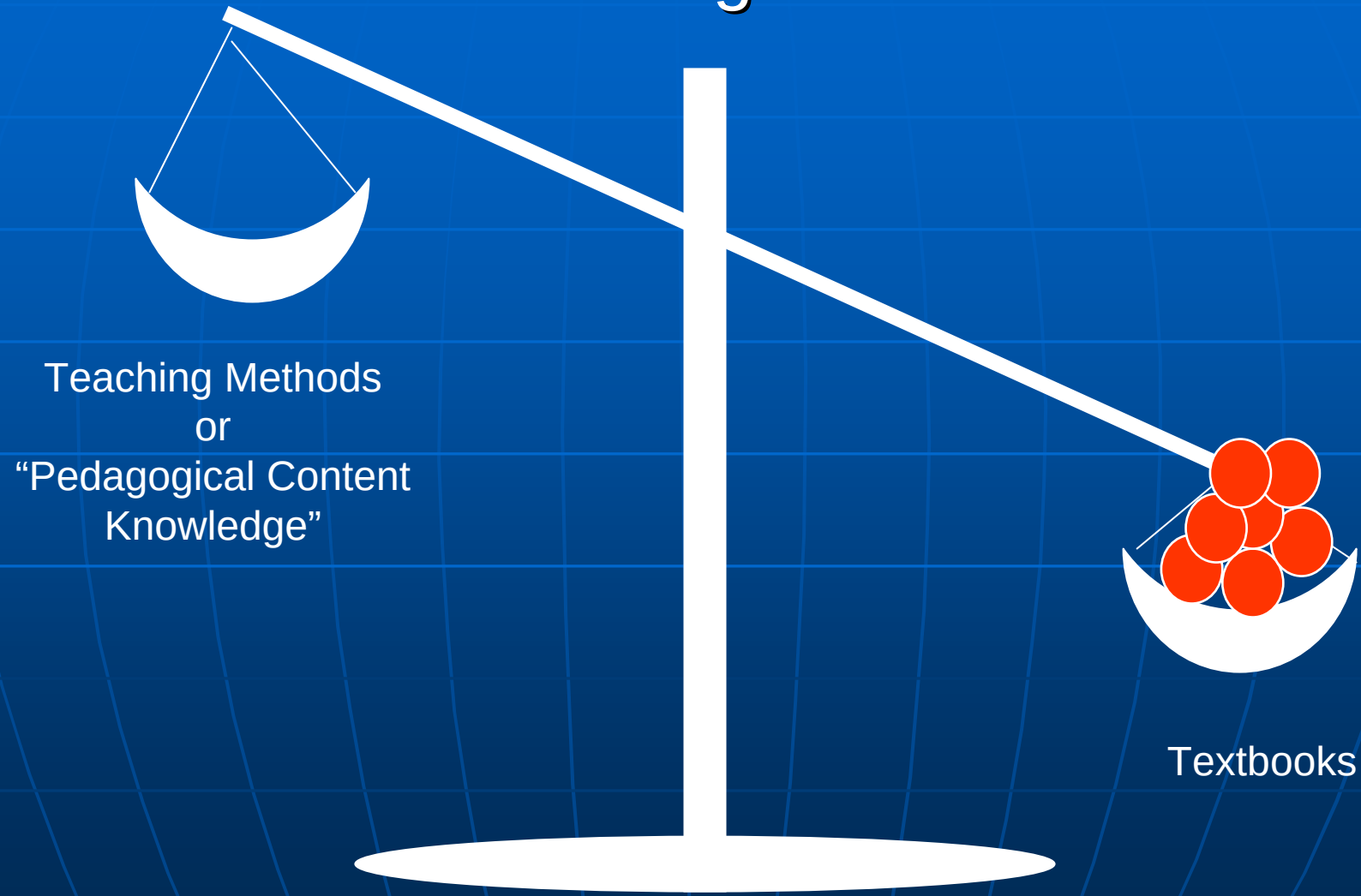
It’s Not Just a Matter of New Materials

In the end, replacing textbooks is easier than replacing teachers.

Walter Doyle

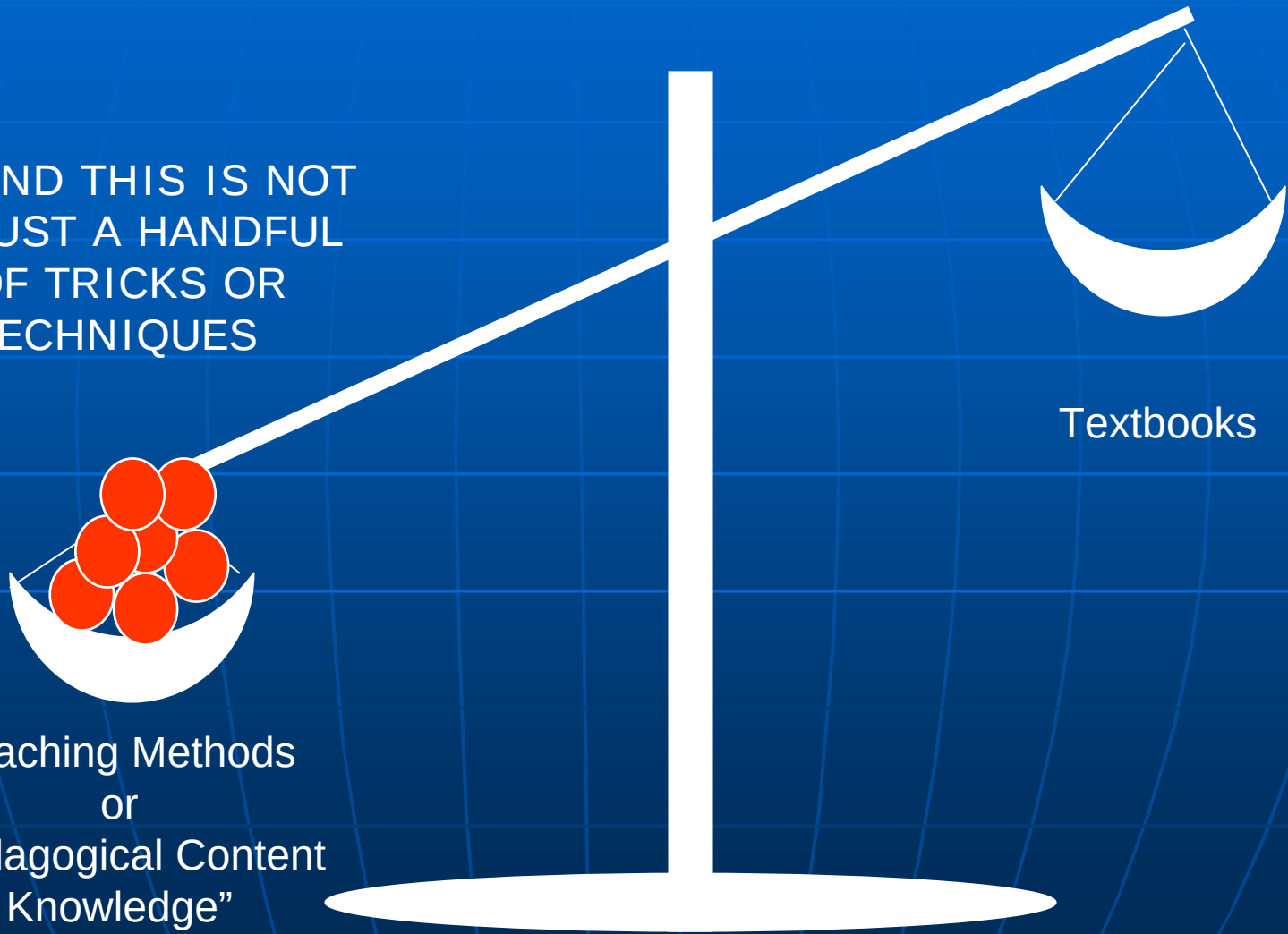


More than Materials: Textbooks and Teaching in the US



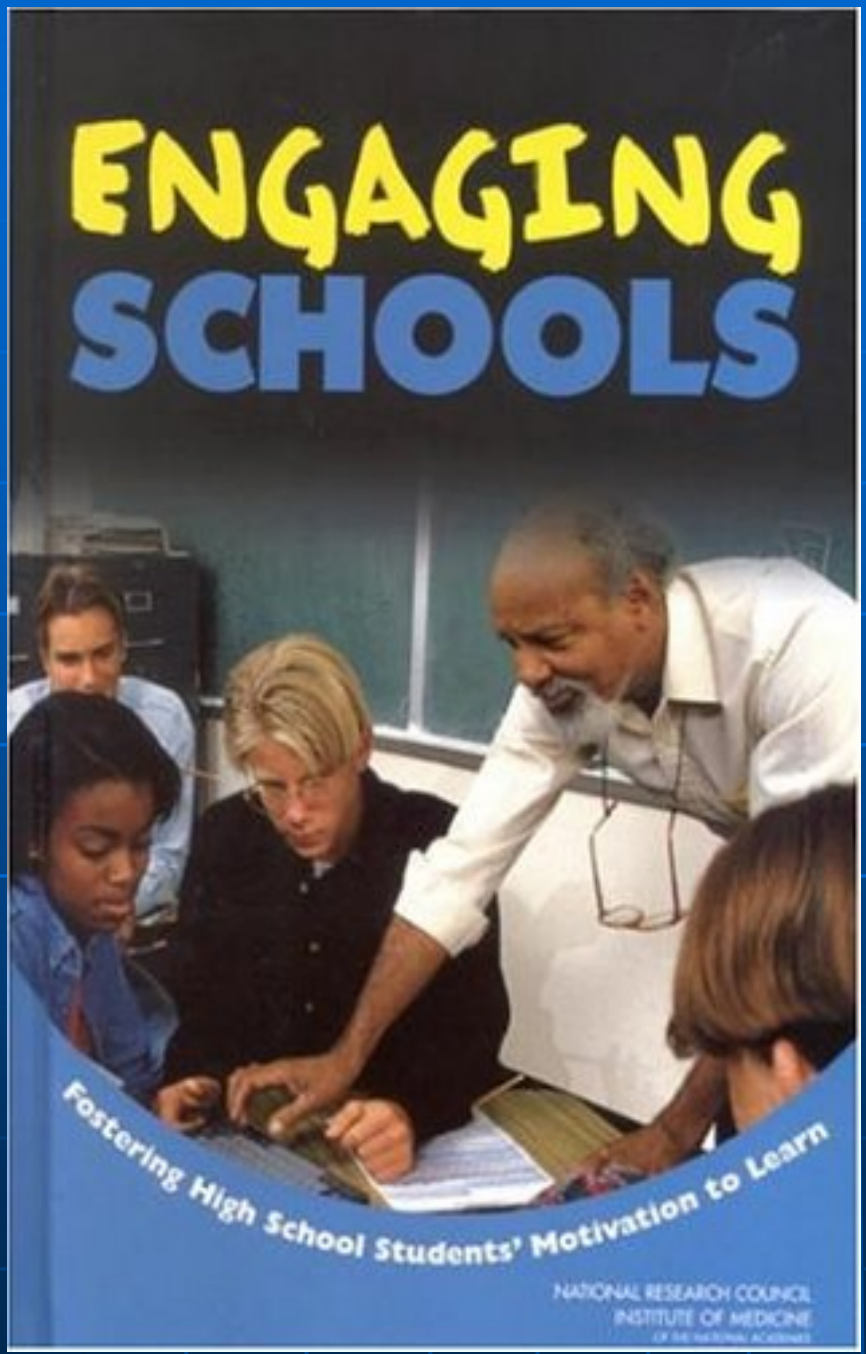
Successful TIMSS Countries

AND THIS IS NOT JUST A HANDFUL OF TRICKS OR TECHNIQUES



Textbooks

Teaching Methods
or
“Pedagogical Content Knowledge”



Motivation

A joint publication by:

National Research Council
Institute of Medicine
of the National Academies

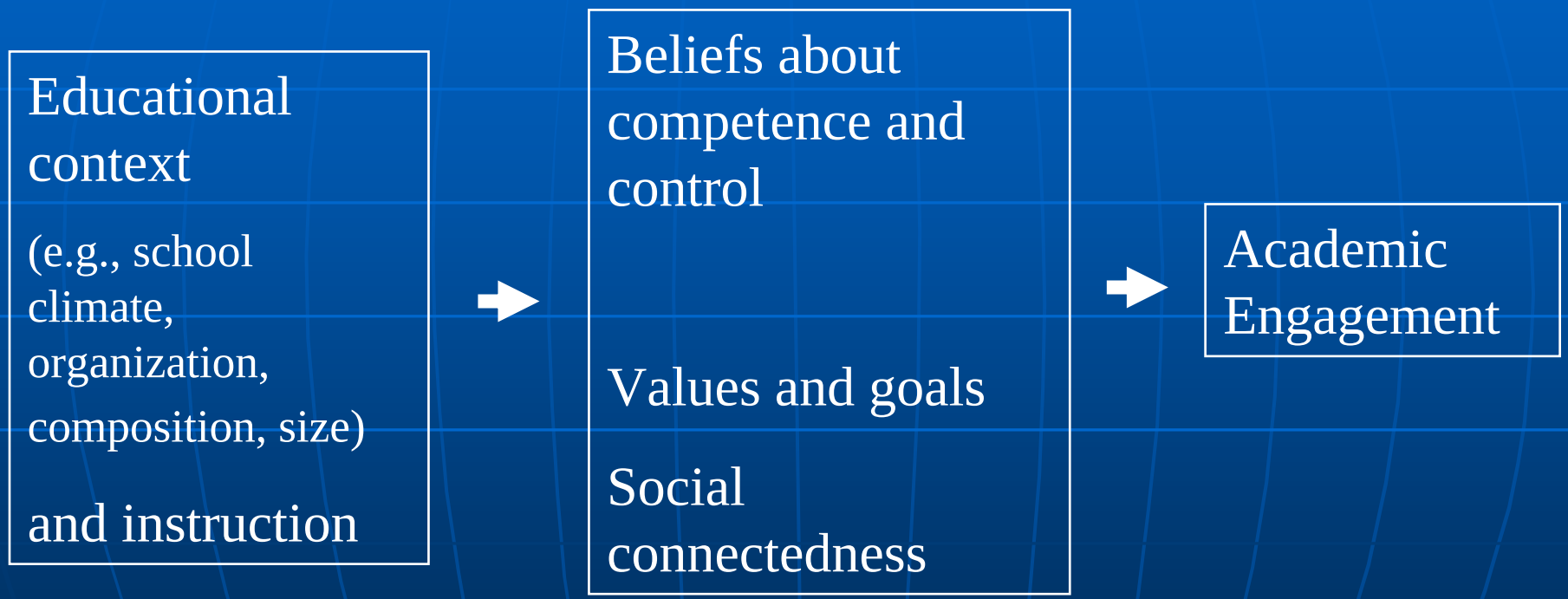
It's a Crisis at Secondary Schools

- On average, 17-year-old African American and Latinos taking the NAEP read and did math about as well as white 13-year-olds
- 40% of 12th graders in central cities scored below basic in math.
- White students in secondary schools have about an 86% chance to get a teacher certified to teach math or science. For nonwhite students it is about 42%

Giving Up

- A survey of 13,000 8th graders
  - 32% dropped out because they could not keep up with school work.
- A survey of 100,000 7th through 11th grade students
  - students from families with low SES and students of color reported *less understanding* of teachers' lessons and comprehension of the material that they read for school.
  - Although *they spent about as much time on homework* as the other students in the same classes, they were much less likely to complete their homework.
- Generally, students drop out because
  - They are socially and academically unengaged
  - They felt no one cared

Engaging Schools



Some Key Points

- Smaller schools (300 – 900) and caring environments
  - A culture centered on learning
- "Academic press" – high standards for all
  - In mathematics classrooms:
    - Variations in tasks
    - An emphasis on understanding
    - Classroom discussions with peers
- Schools with strong links to parents and communities
- Organizational structure and services that address non-academic needs

Some Summary Thoughts

- Put *Many* Special Education Students on a Richer Mathematical Diet
  - The NCTM Standards are a Guide
- Attend to the Characteristics of These Kids
  - Control Procedural Knowledge Tasks
  - Teach Strategies
  - Distribute Practice (not necessarily drill)
  - Conceptual understanding needs to be a *habit of mind*
- Motivation and School Connectedness Are Essential
- Use IEPs to Customize the Needs of the Lowest Ability Students in Special Education

50 Years After Brown vs. Board of Education

Mathematics Education is a Civil Rights Issue

"Today . . . the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy. I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of Black voters in Mississippi was in 1961."

Moses, R. P. (2001). *Radical equations: Math literacy and civil rights*.