Montana K-12 Mathematics Content Standards Framework

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Adopted by the Montana Board of Public Education
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Montana K-12 Mathematics

Content Standards Framework

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Introduction

In 2005 the Montana Board of Public Education initiated the Standards Revision Project to assure Montana citizens that its public schools are providing **all** children of our great state with challenging academic expectations. The Montana Board of Public Education is charged with the responsibility of leading a process of standards revision that meets the following guiding principles.

Revised learning standards which are academic in focus, rigorous but attainable, readily understandable, and designed to measure the progress of students toward meeting them, will lead to the improvement of Montana's schools and a brighter future for our people.

Revised standards must clearly and consistently identify what students should know, understand and be able to do. Parents, educators, and the greater Montana community must be involved in the revision process. Revised standards will provide a framework to help guide local curriculum and instruction, encouraging school districts and teachers to place emphasis on critical areas of learning. In addition, standards should be measured and made known to the Montana public.

With the vital purpose of improving Montana's schools as our goal, the Montana Board of Public Education sets forth the following criteria to guide Standards Revision:

1. Standards will be academic in nature and content specific.
2. Standards will be challenging and rigorous.
3. Standards will be clear, understandable and free of jargon.
4. Standards will be measurable.
5. Standards will address diversity specifically fulfilling the commitment to implementing MCA 20-1-501, Indian Education for All.

With the purpose of developing a successful and useful product, the Montana Board of Public Education sets forth the following process to guide the Montana Standards Revision:

1. Use the existing Montana Standards Framework – current accreditation program delivery and foundation standards, content and performance standards and benchmarks, and existing structure (4^{th}, 8^{th}, and upon graduation);
2. Use proven practices from Montana classrooms;
3. Consider international, national and other states' standards;
4. Consider entrance expectations for workplace and postsecondary education;
5. Consider achievement and other related data;
6. Consider other research e.g., Education Northwest, School Redesign Network, National Study of School Evaluation, etc.;
7. Consider comments from professional education associations;
8. Consider comments from tribal and school district educators;
9. Consider recommendations from the Montana Advisory Council for Indian Education; and
10. Involve the Montana public.

Pursuant to Article X Sect 1(2) of the Constitution of the state of Montana and statutes §20-1-501 and §20-9-309 2(c) MCA, the implementation of these standards must incorporate the distinct and unique cultural heritage of Montana American Indians.

Components of the Mathematics Content Standards Framework

The Mathematics Content Standards Framework is a set of agreements, rationales, and rules that provides the foundation for standards-based Mathematics education in Montana. This framework is the blueprint for further development of key components, such as Essential Learning Expectations, Performance Rubrics, and curriculum. The content standards framework contains:

- K-12 content standards;
- rationale for each content standard;
- benchmarks at end of grade 4, end of grade 8, and upon graduation;
- performance descriptors at the levels of novice, nearing proficiency, proficient and advanced; and
- works cited.

In order to use this framework effectively, it is essential to understand the distinctions between and intended purpose of its various components.

Content Standards: The four mathematics content standards indicate what all students should know, understand, and be able to do in Mathematics. Their purpose is to guide the mathematics curriculum and to communicate the breadth of the mathematics to be taught to all students. A district's mathematics curriculum should be designed so that learning encompasses all four standards.

Rationales: Outlines the fundamental reasons for each of the content standards and provides the basis for the knowledge and skills included in the benchmarks.

Benchmarks: The benchmarks define expectations for students' scientific knowledge and skills along a developmental continuum. They define expectations for proficient students at the end of grade 4, end of grade 8, and upon graduation. Their purpose is to state clearly and specifically what the students should know and be able to do within each content standard. A district's curriculum should include the entire progression of knowledge contained in the benchmarks.

Performance Descriptors: Performance descriptors define how well students apply the knowledge and skills they have acquired. They gauge the level to which benchmarks have been attained in terms of range, frequency, facility, depth, creativity and quality. Achievement of curricular goals is assessed by the performance descriptors.

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Preface to Mathematics Content Standards

The world as we know it is changing at an ever increasing pace. The teaching of mathematics in Montana's public schools needs to be flexible enough to deliver rigorous material that continues to be relevant to the changing lives of our students. In that vein, Montana teachers are challenged to envision the world not as we know it today, but the world our students will be living in tomorrow.

Envision a classroom where instruction is focused on the *big* ideas of mathematics. On a daily basis, students are expected to engage, interact, collaborate, explain and excel. Envision the powerful students such an atmosphere will create—students who are active, excited, curious, and confident; students who *learn*. In this classroom, mathematics is more than just content to be studied; it is an activity to be enjoyed.

There are many aspects of our students' school experience that are outside of our control. However, we do have influence over the mathematics we teach and how we teach it. Montana's mathematics teachers are first class. They are innovators. The standards set forth in this document are of the same quality. To bring them to life requires that Montana educators do what they do best: innovate, challenge and achieve.

Mathematical rigor is an elusive term with multiple meanings. To a pure mathematician, rigor is a mark of excellence. To a K-12 educator, "rigorous" often means "difficult," as in "AP calculus is rigorous." In the Montana Mathematics Content Standards, *rigor* is a process where students:

- approach mathematics with a disposition to accept challenge and apply effort;
- engage in mathematical work that promotes deep knowledge of content, analytical reasoning, and use of appropriate tools; and
- emerge fluent in the language of mathematics, proficient with the tools of mathematics, and empowered as mathematical thinkers.

The Mathematics Standards Development Process

The first efforts to develop and formalize state-level academic content standards were carried out by K-12 educators and largely dependent on intuition and experience. Since then, standards revision processes have evolved as the age of accountability has increased the need for research-based, clearly delineated content standards. Most academic standards now include rationales and incorporate findings from formal research studies and other sources to lend strength and validity to the resulting documents.

In the past, large-scale assessments were primarily used to evaluate the scope and depth of knowledge acquired by students. Today's assessments are also used to determine the effectiveness of curriculum and to hold districts, schools, and teachers accountable for their role in the educational process. Data collected through standardized assessments are used to measure Adequate Yearly Progress (AYP), which can have significant consequences in the life of a school. With this in mind, the 2008-09 Montana Mathematics Content Standards Revision Team worked to develop a clear, concise document, free of jargon, that plainly lays out what is expected of the proficient mathematics student at the end of grade 4, end of grade 8, and upon graduation.

Support for the Montana Mathematics Content Standards: Trends and Philosophies
Students need to be able to enter *tomorrow's* technology-driven global society equipped with the requisite mathematical knowledge and skills essential for success. For some students, this means adequate preparation to pursue higher education; for others, it means the foundation needed to enter a competitive global market with a steep learning curve and limited time for on-the-job training. Regardless of their future trajectory, all Montana students must possess *quantitative literacy* to ensure success in their endeavors.

Quantitative literacy is defined as "the level of mathematical knowledge and skills required of all citizens" (Dossey qtd. in "Why Numbers Count"). Effective mathematics teachers recognize quantitative literacy as a moving target and adapt to the subjective and shifting factors that influence how mathematics is learned and applied. The following discussion addresses these factors, embodied as mathematical processes, mathematical proficiencies, and principles for mathematics education. These fundamental elements interweave with the Montana Mathematics Content Standards like a mathematical knot with no beginning and no end.

Mathematical Processes
The National Council of Teachers of Mathematics Principles and Standards for School Mathematics recognizes five processes that complement and enhance the learning of mathematical content: connections, communication, representation, problem solving, and reasoning. The Office of Public Instruction (OPI) advocates the importance of viewing mathematics through these five lenses because:

- **Mathematics does not exist in isolation.** Learning takes place when students see connections within mathematics and apply their mathematical knowledge to other disciplines and authentic contexts;
- **Mathematics does not follow a single fixed path.** Learning takes place through multiple routes as students visualize, represent, interpret, and construct mathematical ideas in a variety of ways;
- **Mathematics is not a private enterprise.** Learning takes place when students express their mathematical ideas both verbally and in writing, engage in discourse, and work together to build concepts;
- **Mathematics is not free of context.** Learning takes place when students use mathematics to explore ideas, model situations, solve problems, and question and comprehend the world around them; and
- **Mathematics is about doing, not simply knowing.** Learning takes place when students reason, conjecture, reflect, predict, and justify their thinking to themselves and others.

For deep, successful, and lasting learning to take place, all five of these mathematical processes must be embraced and incorporated into the teaching of mathematics. In particular, the OPI values reasoning as a fundamental "habit of mind" for making sense of mathematics. The Montana Mathematics Content Standards reflect this view in the references to reasoning and sense making, emphasizing "doing" mathematics over simply knowing facts, skills and procedures.

Mathematical Proficiency
The National Research Council has identified five research-based building blocks for mathematical proficiency. These are:

- **Conceptual understanding**—comprehension of mathematical concepts, operations, and relations;
- **Procedural fluency**—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- **Strategic competence**—ability to formulate, represent, and solve mathematical problems;
- **Adaptive reasoning**—capacity for logical thought, reflection, explanation, and justification; and
- **Productive disposition**—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (National Research Council: 116)

What does mathematical proficiency mean for Montana? Performance in mathematics is measured both by accuracy and by conceptual understanding. Students know how to recognize a problem, choose appropriate procedures, seek the solution with persistence, and judge their results. Students not only possess a set of mathematical tools, they know what each tool can do and when to use it. Montana students must do mathematics themselves, not simply acknowledge the mathematics done by others. Finally, the study of mathematics must be approached in a way that allows students both to appreciate the value of mathematical competency and to believe they can achieve it themselves.

Principles of Montana Mathematics
The Montana Mathematics Content Standards were conceived and developed under a set of guiding principles agreed upon by all stakeholders in the process. Through high-quality professional development, teachers must embrace these principles and embed them into curriculum planning, instruction, and assessment of mathematics.

- **All students can successfully learn mathematics.** Adopting this view requires teachers to hold high expectations for all their students and to create mathematical experiences that enable success for all.
- **Mathematical processes are fundamental companions to content.** The five processes described earlier are essential to creating an environment where students can acquire, apply, and make meaning of mathematics.
- **Mathematics is a human endeavor with scientific, social, and cultural relevance.** Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians.
- **Technology is integral to learning mathematics.** Today's students are fluent in the language of digital media and technology. Montana educators must maximize technology's potential for enhancing mathematics learning.
- **Mathematics education is for the future, not for today.** To paraphrase a now-famous quote from Karl Fisch (qtd. in Shift Happens) today's students are preparing for jobs that do not yet exist, using technologies that are yet to be invented, to solve problems yet to be identified. Mathematics must be viewed not only through the lens of past experience, but also through a lens that will steer our students through the 21st century.

Implementing the Vision

The Montana Mathematics Content Standards and Performance Descriptors are not about mandating curriculum or recommending specific courses in Montana's schools. Instead, they are about preparing students to work and live successfully in a society that is increasingly technical, global and multicultural. The Board of Public Education has set high expectations for the performance of Montana students at all levels; it is the responsibility of local communities and districts to determine the path for their students to achieve the goals set out in this document.

Number Sense and Operation Content Standard 1

A student, applying reasoning and problem solving, will use number sense and operations to represent numbers in multiple ways, understand relationships among numbers and number systems, make reasonable estimates, and compute fluently within a variety of relevant cultural contexts, including those of Montana American Indians.

Rationale

Number sense and computational fluency are the foundation for school mathematics and life in a multicultural and quantitative society. Students who have a sense of quantity are fluent with basic facts, perform mental computations, understand that knowing the properties of operations help them solve problems determine the reasonableness of solutions, and use number to describe their world. The foundation of number sense and operations supports the other content standards.

Benchmarks

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
1.1 Whole Number Relationships: Demonstrate relationships among whole numbers; identify place value up to 100,000 and compare numbers (e.g., greater than, less than, and equal to).	**1.1 Rational Number Relationships**: Recognize, model, and compare different forms of integers and rational numbers including percents, fractions, decimals, and numbers using exponents and scientific notation.	**1.1 Quantification**: Use multiple notations to perform and interpret the effects of operations on very large and very small numbers with and without technology.
1.2 Estimation and Operations: Estimate sums, differences, products, and quotients when solving problems. Add, subtract, multiply (three-digit by two-digit factors), and divide (two-digit dividends by one-digit divisors) to solve problems. Demonstrate fluency with basic facts.	**1.2 Estimation and Reasonableness**: Select and apply appropriate estimation strategies to judge the reasonableness of solutions to problems including those computed on a calculator. Demonstrate correct use of order of operations.	**1.2 Estimation and Accuracy**: Identify situations where estimation is appropriate and determine the degree of accuracy needed for a given problem situation (and the appropriate precision in which to report answers).

Number Sense and Operation Content Standard 1

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
1.3 Whole Number Concepts: Develop multiplication and division concepts, apply number and operation models and strategies, and reason and justify using properties of operations.	**1.3 Number Theory**: Use number theory concepts such as prime factorization, greatest common factor, and least common multiple in problem situations.	**1.3 Equivalence with Multiple Notation**: Given a representation of a number or expression, find equivalent representations using multiple notations (e.g., $x^{1/2}$ vs. $\sqrt{x}$ and visual representation of multiplying binomials).
1.4 Common Fractions and Decimals: Identify and model common fractions such as, tenths, fourths, thirds, and halves; and decimals such as money and place value to 0.001; and recognize and compare equivalent representations.	**1.4 Rational Number Operations**: Compute fluently and solve multi-step problems using integers, fractions, decimals, and numbers in exponential form.	**1.4 Properties of Numbers and Number Systems**: Analyze and apply the properties of numbers and number systems.
1.5 Length, Time, and Temperature: Select and apply appropriate standard units and tools to measure length, time, and temperature within relevant scientific and cultural situations, including those of Montana American Indians.	**1.5 Metric and Standard Measurement**: Use metric and standard units of measurement in relevant scientific and cultural situations, including those of Montana American Indians, compare and convert within systems, and use appropriate technology.	**1.5 Modeling Relationships and Change**: Identify givens and unknowns in familiar and unfamiliar situations (e.g., finance, culture, including Montana American Indians, and nature) and describe relationships between variables.
	1.6 Proportional Reasoning: Understand and apply proportional relationships to model real world situations and to solve problems involving rates, ratios, proportions, percents, and direct variation.	

Data Analysis Content Standard 2

A student, applying reasoning and problem solving, will use data representation and analysis, simulations, probability, statistics, and statistical methods to evaluate information and make informed decisions within a variety of relevant cultural contexts, including those of Montana American Indians.

Rationale

Data analysis and statistical literacy pertain to all aspects of daily life within multiple cultures. As consumers of information, students who analyze data to make decisions and predictions are better prepared to be responsible citizens. Students who understand and apply basic concepts of probability and make connections to data analysis build strong quantitative reasoning for productive personal and professional lives.

Benchmarks

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
2.1 Representing Data: Collect, represent, and organize data in tables, dot plots, bar graphs, pictographs, and stem-and-leaf plots using technology when appropriate.	**2.1 Representing and Comparing Data**: Collect data from a variety of contexts (e.g., science, history, and culture, including Montana American Indians). Organize and represent data in box plots, scatter plots, histograms, and circle graphs using technology when appropriate.	**2.1 Representing and Analyzing Data**: Select, create, and compare graphical or numerical representations of data sets using technology when appropriate. Reason about distributions using measures of central tendency and spread (e.g., percentiles, quartiles, inter-quartile range, and standard deviation).
2.2 Evaluating Data: Solve problems and make decisions using data descriptors such as minimum, maximum, median, and mode within scientific and cultural contexts, including those of Montana American Indians.	**2.2 Evaluating Data and Making Conjectures**: Interpret, analyze, and evaluate data using mean, median, range, and quartiles to identify trends and make decisions and predictions about data within scientific and cultural contexts, including those of Montana American Indians.	**2.2 Evaluating Validity**: Evaluate the validity of reports based on collected and/or published data by considering the source of the data, the design of the study, and the way data are displayed, analyzed, and interpreted.

Data Analysis Mathematics Content Standard 2

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
2.3 Likelihood of Events: Describe events from multicultural contexts, including those of Montana American Indians, as likely or unlikely and discuss the degree of likelihood using words such as certain, equally likely, and impossible.	**2.3 Finding Probability and Predicting**: Create sample spaces and simulations from events found in different cultures, including those of Montana American Indians, determine experimental and theoretical probabilities, and use probability to make predictions.	**2.3 Rules of Probability and Expected Value**: Make, evaluate, and justify decisions based on probabilities in multicultural situations, including those of Montana American Indians (e.g., finding expected value and using rules of probability).
		2.4 Counting Methods: Use technology as needed to determine the possible number of outcomes for an event or compound event using the fundamental counting principle, permutations, combinations, and other systematic counting methods.
		2.5 Curve Fitting: Model two-variable data using curve fitting with and without technology. Write an equation for a given model and decide when or if predictions based on this equation are valid.

Geometric Reasoning Content Standard 3

A student, applying reasoning and problem solving, will understand geometric properties, spatial relationships, and transformation of shapes, and will use spatial reasoning and geometric models to analyze mathematical situations within a variety of relevant cultural contexts, including those of Montana American Indians.

Rationale

Geometric reasoning complements the study of number, operations, and probability models. Students who have a sense of space analyze two- and three-dimensional shapes and their properties and relationships, and can make connections within mathematics. Geometric reasoning helps students appreciate and value mathematics and make connections to their world through multiple cultural contexts.

Benchmarks

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
3.1 Two-Dimensional Attributes: Describe, compare, and analyze attributes of two-dimensional shapes.	**3.1 Properties of Solids and Figures**: Define, classify and compare properties of solids and plane figures, including lines and angles.	**3.1 Conjectures and Inductive Reasoning**: Formulate and evaluate conjectures about geometric objects and their properties, with and without technology, applying inductive reasoning when appropriate.
3.2 Three-Dimensional Attributes: Describe attributes of three-dimensional shapes such as cubes and other rectangular prisms, pyramids, cylinders, cones, and spheres.	**3.2 Congruence and Similarity**: Use spatial reasoning to determine congruence, similarity, and symmetry of objects in mathematics, art, science, and culture, including Montana American Indians.	**3.2 Applications of Geometric Models**: Use spatial reasoning and geometric models to solve problems with and without technology in the contexts of art, science, and culture, including Montana American Indians.
3.3 Basic Transformations: Use spatial reasoning to identify slides and flips of congruent figures within artistic and cultural contexts, including those of Montana American Indians.	**3.3 Transformations including Dilations**: Define, identify, and execute transformations including translations, rotations, reflections, and dilations with appropriate technology.	**3.3 Multiple Geometric Approaches**: Identify, analyze, and use transformational, coordinate, and synthetic geometric approaches to solve problems.

Geometric Reasoning Content Standard 3

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
3.4 Linear Measurement: Estimate and measure linear attributes of objects in metric units such as centimeters and meters and customary units such as inch, foot, and yard.	**3.4 Angles, Surface Area, and Volume**: Measure and compute angles, perimeter, area, surface area, and volume including the use of formulas and choosing appropriate units.	**3.4 Indirect Measurement**: Determine measures of two- and three-dimensional objects and their elements using trigonometric ratios, proportionality, the Pythagorean Theorem, and angle relationships.
3.5 Area and Perimeter: Define and determine area and perimeter of common polygons using concrete tools such as grid paper, objects, or technology and justify the strategy used.	**3.5 Justifying Relationships**: Develop informal arguments to verify geometric relationships and solve problems such as an informal justification of the Pythagorean Theorem in a variety of contexts.	**3.5 Methods of Proof**: Establish the validity of geometric conjectures using deductive reasoning, indirect proof, and counterexamples, and critique arguments made by others.

Algebraic and Functional Reasoning Content Standard 4

A student, applying reasoning and problem solving, will use algebraic concepts and procedures to understand processes involving number, operation, and variables and will use procedures and function concepts to model the quantitative and functional relationships that describe change within a variety of relevant cultural contexts, including those of Montana American Indians.

Rationale

The study of algebra and functions opens doors and expands opportunities in numerous 21st century careers throughout many cultures. Students who generalize patterns and represent relationships in multiple ways develop significant understandings of mathematics and the use of quantitative reasoning in other disciplines. Algebra and functions are powerful tools for modeling real world relationships and making informed decisions.

Benchmarks

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
4.1 Patterns and Relations: Describe, extend, and make generalizations about geometric or numeric patterns.	**4.1 Representing and Generalizing Patterns**: Create and use tables, graphs or diagrams, symbolic expressions, and verbal descriptions to represent, analyze, and generalize a variety of patterns involving numbers and operations.	**4.1 Representing Functions**: Represent functions in a variety of ways including tables, graphs or diagrams, verbal descriptions, and symbolic expressions in recursive and explicit form. Justify the choice of an appropriate form for solving a given problem.
4.2 Symbols and Expressions: Use letters, boxes, or symbols to represent numbers in simple expressions or equations to demonstrate a basic understanding of variables.	**4.2 Linear Functions**: Identify linear and non-linear functional relationships and contrast their properties using tables, graphs, or equations with appropriate technology.	**4.2 Variables and Parameters**: Determine the appropriate symbolic representation of a given contextual situation (e.g., variables and parameters in equations, inequalities, functions, and matrices).

Algebraic and Functional Reasoning Content Standard 4

A proficient student will:

End of Grade 4	End of Grade 8	Upon Graduation
4.3 Properties of Number and Operation: Use number patterns to investigate properties of numbers such as even or odd and properties of operations such as commutative, associative, distributive, and the multiplicative and additive identities.	**4.3 Multi-step equations and inequalities**: Use number properties and inverse operations to solve multi-step equations and inequalities involving a single variable.	**4.3 Solving Systems of Equations and Inequalities**: Solve a variety of equations, inequalities and systems of equations and inequalities, justify the solution process, and interpret the solution in context.
4.4 Equivalent Expressions: Develop an understanding of equivalence by expressing numbers, measures, and numerical expressions involving operations in a variety of ways.	**4.4 Equivalent Algebraic Expressions**: Recognize, simplify, and generate equivalent forms of algebraic expressions, justifying each step with properties of operations.	**4.4 Families of Functions and Transformations**: Analyze the effects of transformations on families of functions and recognize their characteristics. Represent and use functions in equivalent forms to identify and perform transformations.
4.5 Numerical Modeling with Manipulatives: Model problem situations with manipulatives or technology and use multiple representations such as words, pictures, tables, or graphs to draw conclusions using cultural contexts, including those of Montana American Indians.	**4.5 Linear Modeling**: Identify and compute rate of change/slope and intercepts from equations, graphs, and tables; model and solve contextual problems involving linear proportions or direct variation using cultural contexts, including those of Montana American Indians.	**4.5 Analyzing and Conjecturing with Models**: Given data or a problem situation, select and use an appropriate function model to analyze results or make a prediction with and without technology using cultural contexts, including those of Montana American Indians.

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Montana K-12 Mathematics Performance Descriptors A Profile of Four Levels

The Mathematics Performance Descriptors define how well students perform at four performance levels: advanced, proficient, nearing proficiency, and novice. These profiles describe students as they apply the knowledge and skills defined in the benchmarks and found in the "Benchmarks At-A-Glance" document for End of Grade 4, End of Grade 8, and Upon Graduation.

Advanced	Proficient	Nearing Proficiency	Novice
A student at the advanced level in mathematics demonstrates superior performance. He/she:	A student at the proficient level in mathematics demonstrates solid academic performance. He/she:	A student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficiency. He/she:	A student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental for proficiency. He/she
gives responses that exhibit advanced understanding of the problem or situation presented	gives responses that exhibit clear understanding of the problem or situation presented	gives responses that exhibit some understanding of the problem or situation presented	gives responses that exhibit significant difficulty in understanding basic concepts, reasoning, implementing problem solving strategies, and making connections
consistently demonstrates advanced conceptualization	makes connections within the mathematical and/or real world	makes rudimentary connections between the mathematical and/or real world	severely lacks basic skills, representation, structure, and process development

Advanced	Proficient	Nearing Proficiency	Novice
makes connections within and between the mathematical and real world	determines a process and sufficiently communicates correct problem solving strategies through relevant representations	struggles to communicate effectively	attempts to solve problems
applies more than one process and uses multiple representations to determine solutions accurately	has occasional errors but these do not interfere with appropriate strategies	uses limited evidence of representations to show understanding	has substantial errors
clearly communicates and justifies reasoning and structure of solutions	has reasonable and well-supported solutions	has some basic reasoning skills that are apparent but uses insufficient computational skills and problem solving strategies	lacks communication skills that hinder student's progress
These profiles apply to the "Benchmarks At-A-Glance" document for End of Grade 4, End of Grade 8, and Upon Graduation.		has frequent errors and lack of structure that detract from mathematical knowledge and skills	

Montana Office of Public Instruction Denise Juneau, State Superintendent opi.mt.gov	**Montana K-12 Mathematics Performance Descriptors Benchmarks At-A-Glance**	
End of Grade 4	End of Grade 8	Upon Graduation
1.1 whole number relationships 1.2 estimation and operations 1.3 whole number concepts; 1.4 common fractions and decimals 1.5 length, time, and temperature	1.1 rational number relationships 1.2 estimation and reasonableness 1.3 number theory 1.4 rational number operations 1.5 metric and standard measurement 1.6 proportional reasoning	1.1 quantification 1.2 estimation and accuracy 1.3 equivalence with multiple notation 1.4 properties of numbers and number systems 1.5 modeling relationships and change
2.1 representing data 2.2 evaluating data 2.3 likelihood of events	2.1 representing and comparing data 2.2 evaluating data and making conjectures 2.3 finding probability and predicting	2.1 representing and analyzing data sets 2.2 evaluating validity 2.3 rules of probability and expected value 2.4 counting methods 2.5 curve fitting
3.1 two-dimensional attributes 3.2 three-dimensional attributes 3.3 basic transformations 3.4 linear measurement 3.5 area and perimeter	3.1 properties of solids and figures 3.2 congruence and similarity 3.3 transformations including dilations 3.4 angles, surface area, and volume 3.5 justifying relationships	3.1 conjectures and inductive reasoning 3.2 applications of geometric models 3.3 multiple geometric approaches 3.4 indirect measurement 3.5 methods of proof
4.1 patterns and relations 4.2 symbols and expressions 4.3 properties of number and operation 4.4 equivalent expressions 4.5 numerical modeling with manipulatives	4.1 representing and generalizing patterns 4.2 linear functions 4.3 multi-step equations and inequalities 4.4 equivalent algebraic expressions 4.5 linear modeling	4.1 representing functions 4.2 variables and parameters 4.3 solving systems of equations and inequalities 4.4 families of functions and transformations 4.5 analyzing and conjecturing with models

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