Montana
Office of Public Instruction
Denise Juneau, State Superintendent
opi.mt.gov

A Vision for Montana Mathematics

The world as we know it is changing at an ever increasing pace. The teaching of mathematics in Montana's public schools needs to be flexible enough to deliver rigorous material that continues to be relevant to the changing lives of our students. In that vein, Montana teachers are challenged to envision the world not as we know it today, but the world our students will be living in tomorrow.

Envision a classroom where instruction is focused on the *big* ideas of mathematics. On a daily basis, students are expected to engage, interact, collaborate, explain and excel. Envision the powerful students such an atmosphere will create—students who are active, excited, curious, and confident; students who *learn*. In this classroom, mathematics is more than just content to be studied; it is an activity to be enjoyed.

There are many aspects of our students' school experience that are outside of our control. However, we do have influence over the mathematics we teach and how we teach it. Montana's mathematics teachers are first class. They are innovators. The standards set forth in this document are of the same quality. To bring them to life requires that Montana educators do what they do best: innovate, challenge and achieve.

Mathematical rigor is an elusive term with multiple meanings. To a pure mathematician, rigor is a mark of excellence. To a K-12 educator, "rigorous" often means "difficult," as in "AP calculus is rigorous." In the Montana Mathematics Content Standards, *rigor* is a process where students:

- approach mathematics with a disposition to accept challenge and apply effort;
- engage in mathematical work that promotes deep knowledge of content, analytical reasoning, and use of appropriate tools; and
- emerge fluent in the language of mathematics, proficient with the tools of mathematics, and empowered as mathematical thinkers.

The Standards Development Process

The first efforts to develop and formalize state-level academic content standards were carried out by K-12 educators and largely dependent on intuition and experience. Since then, standards revision processes have evolved as the age of accountability has increased the need for research-based, clearly delineated content standards. Most academic standards now include rationales and incorporate findings from formal research studies and other sources to lend strength and validity to the resulting documents.

In the past, large-scale assessments were primarily used to evaluate the scope and depth of knowledge acquired by students. Today's assessments are also used to determine the effectiveness of curriculum and to hold districts, schools, and teachers accountable for their role in the educational process. Data collected through standardized assessments are

used to measure Adequate Yearly Progress (AYP), which can have significant consequences in the life of a school. With this in mind, the 2008-09 Montana Mathematics Content Standards Revision Team worked to develop a clear, concise document, free of jargon, that plainly lays out what is expected of the proficient mathematics student at the end of grade 4, end of grade 8, and upon graduation.

Support for the Montana Mathematics Content Standards: Trends and Philosophies

Students need to be able to enter *tomorrow's* technology-driven global society equipped with the requisite mathematical knowledge and skills essential for success. For some students, this means adequate preparation to pursue higher education; for others, it means the foundation needed to enter a competitive global market with a steep learning curve and limited time for on-the-job training. Regardless of their future trajectory, all Montana students must possess *quantitative literacy* to ensure success in their endeavors.

Quantitative literacy is defined as "the level of mathematical knowledge and skills required of all citizens" (Dossey qtd. in "Why Numbers Count"). Effective mathematics teachers recognize quantitative literacy as a moving target and adapt to the subjective and shifting factors that influence how mathematics is learned and applied. The following discussion addresses these factors, embodied as mathematical processes, mathematical proficiencies, and principles for mathematics education. These fundamental elements interweave with the Montana Mathematics Content Standards like a mathematical knot with no beginning and no end.

Mathematical Processes

The National Council of Teachers of Mathematics Principles and Standards for School Mathematics recognizes five processes that complement and enhance the learning of mathematical content: connections, communication, representation, problem solving, and reasoning. The Office of Public Instruction (OPI) advocates the importance of viewing mathematics through these five lenses because:

- **Mathematics does not exist in isolation.** Learning takes place when students see connections within mathematics and apply their mathematical knowledge to other disciplines and authentic contexts;
- **Mathematics does not follow a single fixed path.** Learning takes place through multiple routes as students visualize, represent, interpret, and construct mathematical ideas in a variety of ways;
- **Mathematics is not a private enterprise.** Learning takes place when students express their mathematical ideas both verbally and in writing, engage in discourse and work together to build concepts;
- **Mathematics is not free of context.** Learning takes place when students use mathematics to explore ideas, model situations, solve problems, and question and comprehend the world around them; and
- **Mathematics is about doing, not simply knowing.** Learning takes place when students reason, conjecture, reflect, predict, and justify their thinking to themselves and others.

For deep, successful, and lasting learning to take place, all five of these mathematical processes must be embraced and incorporated into the teaching of mathematics. In particular, The OPI values reasoning as a fundamental "habit of mind" for making sense of mathematics. The Montana Mathematics Content Standards reflect this view in their references to reasoning and sense making, emphasizing "doing" mathematics over simply knowing facts, skills and procedures.

Mathematical Proficiency

The National Research Council has identified five research-based building blocks for mathematical proficiency. These are:

- **Conceptual understanding**—comprehension of mathematical concepts, operations, and relations;
- **Procedural fluency**—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- **Strategic competence**—ability to formulate, represent, and solve mathematical problems;
- **Adaptive reasoning**—capacity for logical thought, reflection, explanation, and justification; and
- **Productive disposition**—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (National Research Council 116)

What does mathematical proficiency mean for Montana? Performance in mathematics is measured both by accuracy and by conceptual understanding. Students know how to recognize a problem, choose appropriate procedures, seek the solution with persistence, and judge their results. Students not only possess a set of mathematical tools, they know what each tool can do and when to use it. Montana students must do mathematics themselves, not simply acknowledge the mathematics done by others. Finally, the study of mathematics must be approached in a way that allows students both to appreciate the value of mathematical competency and to believe they can achieve it themselves.

Principles of Montana Mathematics

The Montana Mathematics Content Standards were conceived and developed under a set of guiding principles agreed upon by all stakeholders in the process. Through high-quality professional development, teachers must embrace these principles and embed them into curriculum planning, instruction, and assessment of mathematics.

- **All students can successfully learn mathematics.** Adopting this view requires teachers to hold high expectations for all their students and to create mathematical experiences that enable success for all.
- **Mathematical processes are fundamental companions to content.** The five processes described earlier are essential to creating an environment where students can acquire, apply, and make meaning of mathematics.

- **Mathematics is a human endeavor with scientific, social, and cultural relevance.** Relevant context creates an opportunity for student ownership of the study of mathematics. In Montana, the Constitution pursuant to Article X Sect 1(2) and statutes §20-1-501 and §20-9-309 2(c) MCA, calls for mathematics instruction that incorporates the distinct and unique cultural heritage of Montana American Indians.
- **Technology is integral to learning mathematics.** Today's students are fluent in the language of digital media and technology. Montana educators must maximize technology's potential for enhancing mathematics learning.
- **Mathematics education is for the future, not for today.** To paraphrase a now-famous quote from Karl Fisch (qtd. in Shift Happens) today's students are preparing for jobs that do not yet exist, using technologies that are yet to be invented, to solve problems yet to be identified. Mathematics must be viewed not only through the lens of past experience, but also through a lens that will steer our students through the 21st century.

Components of the Standards Document

In order to use this document effectively, it is essential to understand the distinctions between its various components and their intended purpose.

Content Standards: The four mathematics content standards indicate what all students should know, understand, and be able to do in mathematics. Their purpose is to guide the mathematics curriculum and to communicate the breadth of the mathematics to be taught to all students. A district's curriculum should be designed so learning encompasses all four standards.

Benchmarks: The benchmarks define expectations for students' mathematical knowledge and skills along a developmental continuum. They define expectations for proficient students at the end of grade 4, end of grade 8, and upon graduation. Their purpose is to state clearly and specifically what the students should know and be able to do within each content standard. A district's curriculum should include the entire progression of knowledge contained in the benchmarks.

Performance Descriptors: Performance descriptors define how well students apply acquired knowledge and skills. They gauge the level to which benchmarks have been attained in terms of range, frequency, facility, depth, creativity and quality. Achievement of curricular goals is assessed by the performance descriptors.

Implementing the Vision

The Montana Mathematics Content Standards and Performance Descriptors are not about mandating curriculum or recommending specific courses in Montana's schools. Instead, they are about preparing students to work and live successfully in a society that is increasingly technical, global and multicultural. The Board of Public Education has set high expectations for the performance of Montana students at all levels; it is the responsibility of local communities and districts to determine the path for their students to achieve the goals set out in this document.

Works Cited

Fisch, Karl. Shift Happens. Feb. 2007. YouTube. 2008. 14 Apr. 2009 <http://www.youtube.com>.

National Council of Teachers of Mathematics. Principles and Standards for School Mathematics. 2000. Reston, Va: National Council of Teachers of Mathematics, 2005.

National Research Council. Adding It Up: Helping Children Learn Mathematics. 2001. Ed. J. Kilpatrick, J. Swafford, and B. Findell. Washington, DC: National Academy Press, 2006. 116.

Why Numbers Count: Quantitative Literacy for Tomorrow's America. 1997. Ed. Lynn Arthur Steen. New York: College Board, 2001.

Locate electronic version on www.opi.mt.gov/math/index.html